INMO-2015

1. Let ABC be a right-angled triangle with $\angle B = 90^0$. Let BD be the altitude from B on to AC. Let P, Q and I be the incentres of triangles ABD, CBD and ABC respectively. Show that the circumcentre of the triangle PIQ lies on the hypotenuse AC.

2. For any natural number $n > 1$, write the infinite decimal expansion of $1/n$ (for example, we write $1/2 = 0.4\overline{9}$ as its infinite decimal expansion, not 0.5). Determine the length of the non-periodic part of the (infinite) decimal expansion of $1/n$.

3. Find all real functions f from $\mathbb{R} \to \mathbb{R}$ satisfying the relation $f(x^2 + yf(x)) = xf(x+y)$.

4. There are four basket-ball players A, B, C, D. Initially, the ball is with A. The ball is always passed from one person to a different person. In how many ways can the ball come back to A after **<u>seven</u>** passes? (For example $A \to C \to B \to D \to A \to B \to C \to A$ and $A \to D \to A \to D \to C \to A \to B \to A$ are two ways in which the ball can come back to A after seven passes.)

5. Let ABCD be a convex quadrilateral. Let the diagonals AC and BD intersect in P. Let PE, PF, PG and PH be the altitudes from P on to the sides AB, BC, CD and DA respectively. Show that ABCD has an incircle if and only if $\frac{1}{PE} + \frac{1}{PG} = \frac{1}{PF} + \frac{1}{PH}$.

6. From a set of 11 square integers, show that one can choose 6 numbers $a^2, b^2, c^2, d^2, e^2, f^2$ such that $a^2 + b^2 + c^2 \equiv d^2 + e^2 + f^2 \pmod{12}$.