JEE–Advance 2017 (Paper –II)
Mathematics

Section – I (Maximum Marks: 21)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble (s) corresponding to the correct option is darkened.
  Zero Marks : If none of the bubbles is darkened.
  Negative Marks: –1 In all other cases.

1. If $f: R \to R$ is a twice differentiable function such that f''(x) > 0 for all $x \in R$, and $f\left(\frac{1}{2}\right)=\frac{1}{2}$, f(1) = 1, then

(a) $f'(1) \le 0$ (b) $f'(1) > 1$ (c) $0 < f'(1) \le \frac{1}{2}$ (d) $\frac{1}{2} < f'(1) \le 1$

Ans. (b)

Solution:

f''(x) > 0 for all $x \in R$, f(1/2) = 1/2, f(1) = 1

$\Rightarrow$ f'(x) increases

Let g(x) = f(x) – x, $x \in [1/2, 1]$

Then g'(x) = 0 has atleast one real root in (1/2, 1)

f'(x) = 1 has atleast one real root in (1/2, 1)

Hence f'(x) increases $\Rightarrow$ f'(1) > 1

2. If $y = y(x)$ satisfies the differential equation $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1}dx$, x > 0 and

y(0) = $\sqrt{7}$, then y(256) =

(a) 16 (b) 3 (c) 9 (d) 80

Ans. (b)

Solution:

$$\frac{dy}{dx} = \frac{\left(\sqrt{4+\sqrt{9+x}}\right)^{-1}}{8\sqrt{x}\sqrt{9+\sqrt{x}}}$$

$$dy = \frac{1}{\sqrt{4+\sqrt{9+\sqrt{x}}}}\cdot\frac{1}{\sqrt{9+\sqrt{x}}}\cdot\frac{1}{8\sqrt{x}}dx$$

Let $4+\sqrt{9+\sqrt{x}} = t \Rightarrow \frac{1}{2\sqrt{9+\sqrt{x}}}\times\frac{1}{2\sqrt{x}}dx = dt$

$$\int dy = \int \frac{1}{\sqrt{t}}\cdot\frac{1}{2}dt$$

$$y = \sqrt{t} + c$$

$$y = \sqrt{4+\sqrt{9+\sqrt{x}}} + c$$

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at $x = 0,\ y = \sqrt{7}$

$\Rightarrow\ \sqrt{7} = \sqrt{7} + c \Rightarrow c = 0$

$y = \sqrt{4 + \sqrt{9 + \sqrt{x}}}$

at x = 256 $\quad \Rightarrow\ y = \sqrt{4 + \sqrt{9 + \sqrt{256}}} = 3$

3. How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 5?

(a) 198 (b) 162 (c) 126 (d) 135

Ans. (a)

Solution:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 5$

Case-I : Five (1's) and four (0's)

$^9C_5 = 126$

Case-II: One (2) and one (1)

$^9C_2 \times 2! = 72$

$\therefore$ Total = 198

4. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is

(a) $\frac{1}{2}$ (b) $\frac{36}{55}$ (c) $\frac{6}{11}$ (d) $\frac{5}{11}$

Ans. (c)

Solution:

x + y + z = 10

Total number of non-negative solutions = $^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

Now Let z = 2n.

x + y + 2n = 10 ; $n \ge 0$

Total number of non-negative solutions = 11 + 9 + 7+ 5 + 3 + 1 = 36

Required probability = $\frac{36}{66} = \frac{6}{11}$

5. Let S = {1, 2, 3, …, 9}. For k = 1, 2, ….., 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5$ =

(a) 210 (b) 252 (c) 126 (d) 125

Ans. (c)

Solution:

$N_1 = {}^5C_1 . {}^4C_4 = 5$

$N_2 = {}^5C_2 . {}^4C_3 = 40$

$N_3 = {}^5C_3 . {}^4C_2 = 60$

$N_4 = {}^5C_4 . {}^4C_1 = 20$

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$N_5 = {}^5C_5 . {}^4C_0 = 1$

$\therefore$ Total = 126

6. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

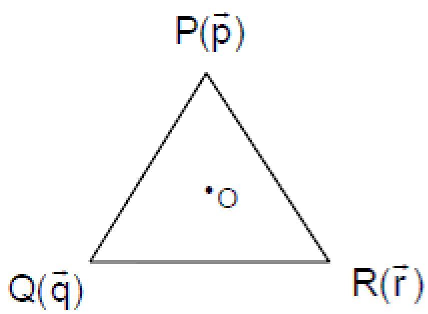
$\overrightarrow{OP}.\overrightarrow{OQ} + \overrightarrow{OR}.\overrightarrow{OS} = \overrightarrow{OR}.\overrightarrow{OP} + \overrightarrow{OQ}.\overrightarrow{OS} = \overrightarrow{OQ}.\overrightarrow{OR} + \overrightarrow{OP}.\overrightarrow{OS}$

Then the triangle PQR has S as its

(a)centroid (b) orthocenter (c) incentre (d) circumcenter

Ans. (b)

Solution:



$\vec{p}.\vec{q} + \vec{r}.\vec{s} = \vec{r}.\vec{p} + \vec{q}.\vec{s} + \vec{q}.\vec{r} + \vec{p}.\vec{s}$

$\Rightarrow \vec{p}.\left(\vec{q} - \vec{r}\right) - \vec{s}.\left(\vec{q} - \vec{r}\right) = 0 \Rightarrow \overrightarrow{PS}.\overrightarrow{QR} = 0$

Similarly $\overrightarrow{PQ}.\overrightarrow{SR} = 0$

$\Rightarrow$ S is orthocentre of the triangle

7. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7, is

(a) 14x + 2y – 15z = 1 (b) –14x + 2y + 15z = 3

(c) 14x – 2y + 15z = 27 (d) 14x + 2y + 15z = 31

Ans. (d)

Solution:

Let plane be

a(x – 1) + b(y – 1) +c(z – 1) = 0

Now, direction ratio of its normal = $\begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & 1 & -2 \\ 3 & -6 & -2 \end{vmatrix} = \hat{i}\left(-14\right) - \hat{j}\left(2\right) + k\left(-15\right)$

So, –14(x – 1) – 2(y – 1) – 15(z – 1) = 0

14x + 2y + 15z = 31

Section – 2 : (Maximum Marks: 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option (s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option (s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble (s) corresponding to all the correct option (s) is(are) darkened.

Partial Marks : + 1 For darkening a bubble corresponding to each correct option, provided NO

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incorrect option is darkened.

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks: – 2 In all other cases.

- For example, if (A), (C) and (D) are the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks and darkening (A) and (B) will get – 2 marks, as a wrong option is also darkened.

8. If : R $\to$ R is a differentiable function such that f'(x) > 2f(x) for all $x \in R$, and f(0) = 1, then

(a) $f(x) > e^{2x}$ in $(0, \infty)$ (b) $f'(x) < e^{2x}$ in $(0, \infty)$

(c) f(x) is increasing in $(0, \infty)$ (d) f(x) is decreasing in $(0, \infty)$

Ans. (a, c)

Solution:

f'(x) – 2f(x) > 0

$\Rightarrow \quad \frac{d}{dx}\left(f(x).e^{-2x}\right) > 0 \quad \Rightarrow$ g(x) = f(x). e–2x is an increasing function.

for x > 0, g(x) > g(0)

$\Rightarrow$ f(x). $e^{-2x} > 1$ $\Rightarrow$ f(x) > e^{2x}

Now f'(x) > 2f(x) > 2.e^{2x}

$\therefore$ f(x) is an increasing function

9. If $I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)} dx$, then

(a) I > $\log_e 99$ (b) $I < \log_e 99$ (c) $I < \frac{49}{50}$ (d) $I > \frac{49}{50}$

Ans. (b, d)

Solution:

Put x – k = p

$$I = \sum_{k=1}^{98} \int_0^1 \frac{k+1}{(k+p)(k+p+1)} dp$$

$$I > \sum_{k=1}^{98} \int_0^1 \frac{k+1}{(k+p+1)^2} dp$$

$$I > \sum_{k=1}^{98} (k+1)\left(\frac{-1}{(k+p+1)}\right)_0^1$$

$$I > \sum_{k=1}^{98} (k+1)\left(\frac{1}{k+1} - \frac{1}{k+2}\right)$$

$$I > \sum_{k=1}^{98} \frac{1}{k+2} = \frac{1}{3} + \ldots\ldots\ldots + \frac{1}{100}$$

$$I > \frac{1}{100} + \ldots\ldots + \frac{1}{100} = \frac{98}{100}$$

$$I > \frac{49}{50}$$

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$$\sum_{k=1}^{98}\int_k^{k+1}\frac{k+1}{x(x+1)}dx$$

$$\frac{k+1}{x(x+1)}<\frac{k+1}{x(k+1)} \quad (\because \text{ least value of } x + 1 \text{ is } k + 1)$$

$$\Rightarrow \frac{k+1}{x(x+1)}<\frac{1}{x}$$

$$\Rightarrow I<\sum_{k=1}^{98}\int_k^{k+1}\frac{1}{x}dx$$

$$\Rightarrow I<\sum_{k=1}^{98}\ell n(k+1)-\ell nk \Rightarrow I<\ell n99$$

10. If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in R^2 : x^3 \le y \le x, 0 \le x \le 1\}$ into two equal parts, then

(a) $2\alpha^4-4\alpha^2+1=0$ (b) $\alpha^4+4\alpha^2-1=0$ (c) $\frac{1}{2}<\alpha<1$ (d) $0<\alpha\le\frac{1}{2}$

Ans. (a, c)

Solution:

$y = x^3$

$$\int_0^1(x-x^3)dx=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

$$\int_0^\alpha(x-x^3)dx=\frac{1}{8}$$

$$4\alpha^2-2\alpha^4=1$$

$$2\alpha^4-4\alpha^4=1$$

$$2t^2-4t+1=0 \quad (\text{taking } = \alpha^2)$$

$$t=\frac{4\pm\sqrt{16-8}}{4}$$

$$t=\frac{4\pm2\sqrt{2}}{4}$$

$$t=\alpha^2=1\pm\frac{1}{\sqrt{2}}$$

$$\therefore \alpha^2=1-\frac{1}{\sqrt{2}} \Rightarrow \frac{1}{2}<\alpha<1$$

11. Let α and β be nonzero real numbers such that $2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta=1$. Then which of the following is/are true?

(a) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$ (b) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(c) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$ (d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

Ans. (b, c)

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Solution:

$\cos\alpha = \left(\frac{1-a}{1+a}\right)$; $a = \tan^2\frac{\alpha}{2}$

$\cos\beta = \left(\frac{1-b}{1+b}\right)$; $b = \tan^2\frac{\beta}{2}$

$$2\left(\left(\frac{1-b}{1+b}\right)-\left(\frac{1-a}{1+a}\right)\right)+\left(\left(\frac{1-a}{1+a}\right)\left(\frac{1-b}{1+b}\right)\right)=1$$

$\Rightarrow$ 2((1 – b) (1 + a) – (1 – a) (1 + b)) + (1 – a) (1 – b) = (1 + a) (1 + b)

$\Rightarrow$ 2(1 + a – b – ab – (1 + b – a – ab)) + 1 – a – b + ab = 1 + a + b + ab

$\Rightarrow$ 4(a – b) = 2(a + b)

$\Rightarrow$ 2a – 2b = a + b

$\Rightarrow$ a = 3b

$$\tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{3}\tan\left(\frac{\beta}{2}\right)$$

12. Let f(x) = $\frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$. Then

(a) $\lim_{x\to1^+} f(x) = 0$ (b) $\lim_{x\to1^-} f(x)$ does not exist

(c) $\lim_{x\to1^-} f(x) = 0$ (d) $\lim_{x\to1^+} f(x)$ does not exist

Ans. (c, d)

Solution:

$$f(1^+) = \lim_{h\to0}\frac{1-(1+h)(1+h)}{h}\cos\frac{1}{h}$$

$$= \lim_{h\to0}\frac{1-(1+h)^2}{h}\cos\frac{1}{h}$$

$$= \lim_{h\to0}\frac{-h^2-2h}{h}\cos\frac{1}{h}$$

$$= \lim_{h\to0}(-h-2)\cos\frac{1}{h}$$

$\Rightarrow$ $\lim_{h\to0} f(1^+)$ does not exist

$$f(1^-) = \lim_{h\to0}\frac{1(1-h)(1+h)}{h}\cos\frac{1}{h}$$

$$= \lim_{h\to0}\frac{1-(1-h^2)}{h}\cos\frac{1}{h}$$

$$= \lim_{h\to0}\frac{h^2}{h}\cos\frac{1}{h}$$

$$= \lim_{h\to0} h\cos\frac{1}{h} = 0$$

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13. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)dt$, then

(a) $g'\left(-\frac{\pi}{2}\right) = 2\pi$ (b) $g'\left(-\frac{\pi}{2}\right) = -2\pi$ (c) $g'\left(\frac{\pi}{2}\right) = 2\pi$ (d) $g'\left(\frac{\pi}{2}\right) = -2\pi$

Ans. (BONUS)

Solution:

$$g(x) = \int_{\sin x}^{\sin 2x} \sin^{-1}(t)dt$$

$g'(x) = \sin^{-1}(\sin 2x).\cos 2x.2 - \sin^{-1}(\sin x).\cos x$

$= 2\cos 2x.\sin^{-1}(\sin 2x) - \cos x.\sin^{-1}(\sin x)$

$$g'\left(-\frac{\pi}{2}\right) = 2\cos(-\pi)\sin^{-1}(\sin(-\pi)) - \cos\left(-\frac{\pi}{2}\right).\sin^{-1}(\sin\left(-\frac{\pi}{2}\right)) = 0$$

$$g'\left(\frac{\pi}{2}\right) = 2\cos(\pi)\sin^{-1}(\sin(\pi)) - \cos\left(\frac{\pi}{2}\right).\sin^{-1}(\sin\left(\frac{\pi}{2}\right)) = 0$$

14. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

(a) f(x) attains its minimum at x = 0

(b) f(x) attains its maximum at x = 0

(c) f′(x) = 0 at more than three points in $(-\pi, \pi)$

(d) f′(x) = 0 at exactly three points in $(-\pi, \pi)$

Ans. (b, c)

Solution:

$$f(x) = \begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$$

$= \cos 2x - \cos 2x(-\cos^2 x + \sin^2 x) + \sin 2x(-2\sin x\cos x)$

$f(x) = \cos 4x + \cos 2x$

$\therefore$ $f(x) = 2\cos^2 2x + \cos 2x - 1$

Let $\cos 2x = t$

$\Rightarrow$ $f(x) = 2t^2 + t - 1$ and $t \in [-1, 1]$

f(x) attains its minima at $t = -\frac{1}{4} \in [-1, 1]$

$f(x), t = -\frac{1}{4} \in [-1, 1]$

$\therefore$ $f(x)\big|_{min} = \frac{2}{16} - \frac{1}{4} - 1 = \frac{-9}{8}$

$\therefore$ $f(x)\big|_{max}$ = 2 + 1 – 1 = 2 ....(when cos 2x = 1)

$f'(x) = -4\sin 4x - 2\sin 2x$

$f'(x) = 0 \Rightarrow 4\sin 4x + 2\sin 2x = 0$

$\Rightarrow 8\sin 2x\cos 2x + 2\sin 2x = 0$

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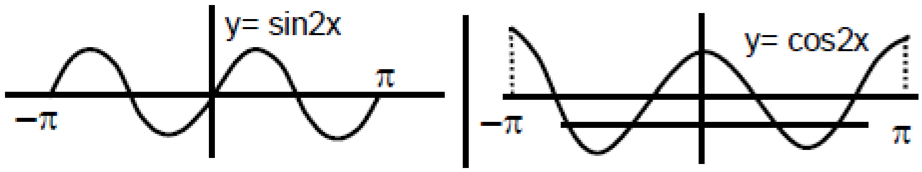
$\Rightarrow$ 2 sin 2x(4 cos 2x + 1) = 0 $\Rightarrow$ sin 2x = 0 or cos 2x = $-\frac{1}{4}$



Hence option (b, c)

Section – 3 : (Maximum Marks: 12)

- This section contains TWO paragraphs.
- Based on each paragraph, there are TWO questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks: +3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks: 0 In all other cases.

PARAGRAPH 1

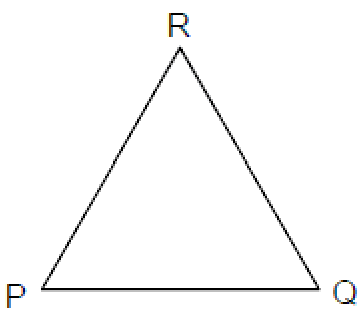
Let O be the origin, and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$, respectively, of a triangle PQR.

15. If the triangle PQR varies, then the minimum value of cos (P + Q) + cos (Q + R) + cos (R +P) is

(a) $-\frac{3}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{3}$ (d) $-\frac{5}{3}$

Ans. (a)

Solution:



cos (P + Q) + cos (Q + R) + cos (R + P) = – cos R – cos P – cos Q

In any Δ, max of cos P + cos Q + cos R = $\frac{3}{2}$

So minimum value of the given expression is $-\frac{3}{2}$

16. $\left|\overrightarrow{OX} \times \overrightarrow{OY}\right| =$

(a) sin (P + Q) (b) sin (P + R) (c) sin (Q + R) (d) sin 2R

Ans. (a)

Solution:

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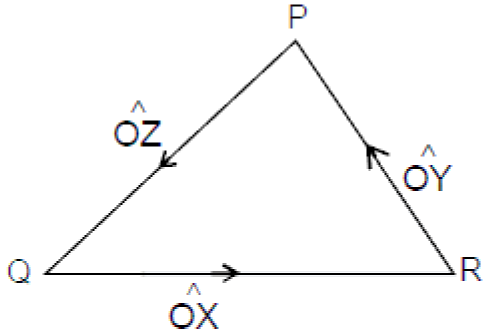

$\cos R = -OX.\,OY$

$\Rightarrow |\cos R| = |OX.\,OY|$

$|OX \times OY| = |\sin R| = |\sin(\pi - (P+Q))| = |\sin(P+Q)| = \sin(P+Q)$

PARAGRAPH 2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$ where $\alpha \neq \beta$. For n = 0, 1, 2, ..., let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then a = 0 = b.

17. $a_{12} =$

(a) $a_{11} + 2a_{10}$ (b) $2a_{11} + a_{10}$ (c) $a_{11} - a_{10}$ (d) $a_{11} + a_{10}$

Ans. (d)

Solution:

As α and β are roots of equation $x^2 - x - 1 = 0$, we get:

$\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^2 = \alpha + 1$

$\beta^2 - \beta - 1 = 0 \Rightarrow \beta^2 = \beta + 1$

$\therefore\ a_{11} + a_{10} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10}$

$= p\alpha^{10}(\alpha+1) + q\beta^{11}(\beta+1)$

$= p\alpha^{10} \times \alpha^2 + q\beta^{10} \times \beta^2$

$= p\alpha^{12} + q\beta^{12}$

$= a_{12}$

18. If $a_4 = 28$, then $p + 2q =$

(a) 14   (b) 7   (c) 21   (d) 12

Ans. (d)

Solution:

$a_{n+2} = a_{n+1} + a_n$

$a_4 = a_3 + a_2 = 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p+q)$

As $\alpha = \dfrac{1+\sqrt{5}}{2}$, $\beta = \dfrac{1-\sqrt{5}}{2}$, we get

$a_4 = 3p\left(\dfrac{1+\sqrt{5}}{2}\right) + 3q\left(\dfrac{1-\sqrt{5}}{2}\right) + 2p + 2q = 28$

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$\Rightarrow \left(\frac{3p}{2}+\frac{3q}{2}+2p+2q-28\right)=0$ ...(i)

and $\frac{3p}{2}-\frac{3q}{2}=0$ ...(ii)

$\Rightarrow$ p = q (from (ii)

$\Rightarrow$ 7p = 28 (from (i) and (ii))

$\Rightarrow$ p = 4

$\Rightarrow$ q = 4

$\Rightarrow$ p + 2q = 12

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