JEE–Advance 2017 (Paper –I)
Mathematics

Section – I (Maximum Marks: 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option (s) is (are) correct.
- For each question, darken the bubble (s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4	If only the bubble (s) corresponding to all the correct option (s) is (are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding to each correct option, provided No incorrect option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get + 4 marks; darkening only (A) and (D) will get + 2 marks and darkening (A) and (B) will get – 2 marks, as a wrong option is also darkened.

1. Let X and Y be two events such that P(X) = $\frac{1}{3}$, $P(X|Y)=\frac{1}{2}$ and $P(Y|X)=\frac{2}{5}$. Then

(a) P(Y) = $\frac{4}{15}$ (b) $P(X'|Y)=\frac{1}{2}$ (c) $P(X\cup Y)=\frac{2}{5}$ (d) $P(X\cap Y)=\frac{1}{5}$

Ans. (a, b)

Solution:

$$\frac{P(X\cap Y)}{P(Y)}=\frac{1}{2}$$

$$\frac{P(Y\cap X)}{P(X)}=\frac{2}{5}$$

$$P(X\cap Y)=\frac{P(Y)}{2}=\frac{2}{5}P(X)=\frac{2}{5}.\frac{1}{3}=\frac{2}{15}\Rightarrow P(Y)=\frac{4}{15}$$

$$\frac{P(\overline{X}\cap Y)}{P(Y)}=\frac{P(Y)-P(X\cap Y)}{P(Y)}=\frac{\frac{4}{15}-\frac{2}{15}}{\frac{4}{15}}=\frac{2}{4}=\frac{1}{2}$$

$$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)=\frac{1}{3}+\frac{4}{15}-\frac{2}{15}=\frac{7}{15}$$

2. Let f: R → (0, 1) be a continuous function. Then, which of the following function (s) has (have) the value zero at some point in the interval (0, 1)?

(a) $e^x-\int_0^x f(t)\sin t\,dt$ (b) $f(x)+\int_0^{\frac{\pi}{2}} f(t)\sin t\,dt$

(c) $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$ (d) $x^9 - f(x)$

Ans. (c, d)

Solution:

$e^x \in (1,e)$ for $x \in (0,1)$ and $0 < \int_0^{\frac{\pi}{2}} f(t)\sin t\,dt < 1$ in $(0,1)$ $\Rightarrow$ (A) is wrong

and $f(x) + \int_0^{\frac{\pi}{2}} f(t)\sin t\,dt > 0$ $\Rightarrow$ (B) is wrong

Let $g(x) = x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t\,dt$ $\Rightarrow$ $g(0) = -\int_0^{\frac{\pi}{2}} f(t)\cot t\,dt < 0$

$g(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t)\cos t\,dt > 0$ $\Rightarrow$ (C) is correct

Let $h(x) = x^9 - f(x)$

$h(0) = -f(0) < 0$

$h(1) = 1 - f(1) > 0$ $\Rightarrow$ (D) is correct

3. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is (are) possible value (s) of x?

(a) $1-\sqrt{1+y^2}$ (b) $-1-\sqrt{1-y^2}$ (c) $1+\sqrt{1+y^2}$ (d) $-1+\sqrt{1-y^2}$

Ans. (b, d)

Solution:

$$\frac{a(x+iy)+b}{x+iy+1} = \frac{ax+b+iay}{x+1+iy} \times \frac{(x+1)-iy}{(x+1)-iy} = \frac{(ax+b)(x+1)+ay^2}{(x+1)^2+y^2} + \frac{i\left(ay(x+1)-y(ax+b)\right)}{(x+1)^2+y^2}$$

$$\Rightarrow \frac{ay(x+1)-y(x+b)}{(x+1)^2+y^2} = y \quad \Rightarrow \quad \frac{ay-by}{(x+1)^2+y^2} = y \qquad (\because a-b=1,\ y \neq 0)$$

$$\Rightarrow (x+1)^2+y^2 = 1 \quad \Rightarrow \quad x+1 = \pm\sqrt{1-y^2} \quad \Rightarrow \quad x = -1 \pm \sqrt{1-y^2}$$

4. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be Sides of a right angled triangle?

(a) a, 4, 1 (b) 2a, 4, 1 (c) a, 4, 2 (d) 2a, 8, 1

Ans. (a, c, d)

Solution:

$y = 2x + 1$ is tangent to $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$

$c^2 = a^2m^2 - b^2$

$1 = 4a^2 - 16 \Rightarrow a^2 = \frac{17}{4}$

[check if $p^2 = q^2 + r^2$]

5. Let [x] be the greatest integer less than or equals to x. Then, at which of the following point(s) the function f(x) = x cos (π (x + [x])) is discontinuous?

(a) x = – 1 (b) x = 1 (c) x = 0 (d) x = 2

Ans. (a, b, d)

Solution:

f(x) = xcos(π (x + [x])

Check continuity at x = n

f(n) = ncos2n π = n

$f(n^+)$ = ncos 2n π = n

$f(n^-)$ = ncos (2n – 1) π = – n

It is discontinuous at all integer points except 0

6. Which of the following is(are) NOT the square of a 3 $\times$ 3 matrix with real entries?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ans. (a, c)

Solution:

$A = B^2 \Rightarrow |A| = |B|^2 = +ve$

(a) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$ = 1(–1) = negative

Matrix B cannot be possible

(b) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$ = 1(1 – 0) = positive

Matrix B can be possible

Ex. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$

(c) $\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$ = – 1 = negative

Matrix B cannot be possible

(D) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ = 1 = positive

Matrix B can be I

7. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and midpoint (h, k), then which of the following is(are) possible value (s) of p, h and k?

(a) p = –1, h = 1, k = – 3 (b) p = 2, h = 3, k = – 4

(c) p = – 2, h = 2, k = – 4 (d) p = 5, h = 4, k = – 3

Ans. (b)

Solution:

$8x - ky + (k^2 - 8h) = 0$

$2x + y - p = 0$

Comparing coefficients of x, y and constant term, we get

$$4 = -k = \frac{k^2 - 8h}{-p}$$

k = – 4

16 – 8h = – 4p

4 – 2h = – p $\Rightarrow$ p = 2h – 4

Section – II (Maximum Marks: 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : + 3 If only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 In all other cases.

8. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$

Ans. (a)

Solution:

D = 0

$$\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} (1+\alpha+\alpha^2) & (2\alpha+1) & (\alpha^2+\alpha+1) \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} (1+\alpha+\alpha^2) & (2\alpha+1) & 0 \\ \alpha & 1 & 0 \\ \alpha^2 & \alpha & 1-\alpha^1 \end{vmatrix} = 0 \Rightarrow (1-\alpha^2)(1+\alpha+\alpha^2-2\alpha^2-\alpha)=0 \Rightarrow (1=-\alpha^2)=0$$

$\alpha = -1$ or 1

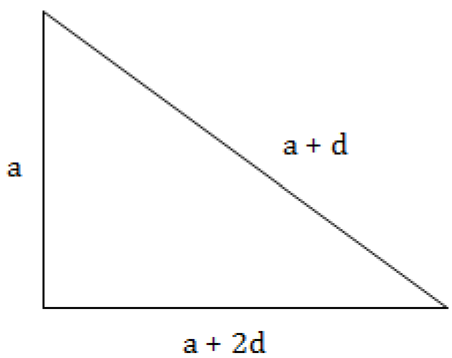
for $\alpha = 1$, system of linear equations has no solution

$\therefore\ \alpha = -1$ so $1+\alpha+\alpha^2 = 1$

9. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

Ans. (6)

Solution:



$\frac{1}{2}a(a+d) = 24$ $\Rightarrow$ a(a + d) = 48 ...(1)

$a^2 + (a+d)^2 = (a+d)^2$ $\Rightarrow$ $3d^2 + 2ad - a^2 = 0$

$(3d - a)(a + d) = 0$

$\Rightarrow$ $3d = a\ (\because\ a + d \neq 0)$

$\Rightarrow$ d = 2

a = 6

so smallest side = 6

10. Let f: $R \to R$ be a differentiable function such that f(0) = 0, $f\left(\frac{\pi}{2}\right) = 3$ and f′(0) = 1. If

$g(x) = \int_x^{\frac{\pi}{2}} [f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t\ f(t)]\, dt$ for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x) =$

Ans. (b)

Solution:

$$g(x) = \int_x^{\pi/2} \frac{d}{dt}(f(t)\operatorname{cosec} t)\, dt$$

$$g(x) = f\left(\frac{\pi}{2}\right)\operatorname{cosec}\left(\frac{\pi}{2}\right) - f(x)\operatorname{cosec} x$$

g(x) = 3 – f(x) cosec x

$$g(x) = 3 - \frac{f(x)}{\sin x}$$

$$\lim_{x\to 0} g(x) = 3 - \lim_{x\to 0}\frac{f(x)}{\sin x}$$

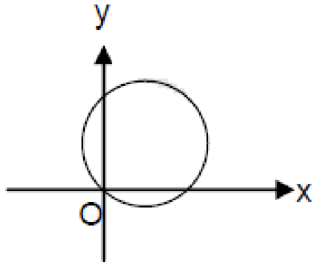
$$= 3 - \lim_{x\to 0}\frac{f'(x)}{\cos x} = 3 - \frac{1}{1} = 2$$

11. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

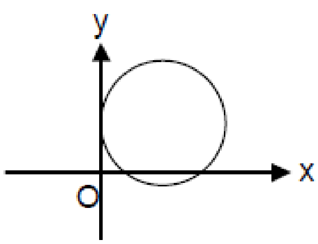
Ans. (b)

Solution:

case-I Passing through origin $\Rightarrow$ p= 0



Cass-II Touches y-axis and cuts x-axis



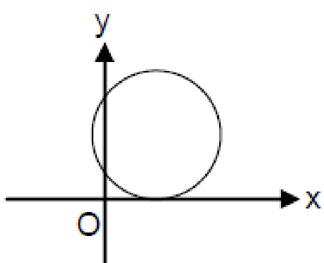
$f^2 - c = 0$ & $g^2 - c > 0$

$4 + p = 0$ $\quad$ $1 + p > 0$

$p = -4$

Not possible

Case – III Touches x-axis and cuts y-axis



$f^2 - c > 0$ & $\quad$ $g^2 - c = 0$

$4 + p > 0$ $\quad$ $1 + p = 0$

So two value of p are possible

12. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x}$ =

Ans. (5)

Solution:

A, B, C, D, E, F, G, H, I, J

$x = 10!$

$y = {}^{10}C_1,\ {}^{10}C_2.\ 8!\ {}^{9}C_8$

$$\frac{y}{9x} = \frac{{}^{10}C_1.{}^{10}C_2.8!\,9}{9\times 10!} = \frac{10!\times 45}{9\times 10!} = 5$$

Section – III (Maximum Marks: 18)

- This section contains **SIX** questions.
- This section contains **TWO** tables (each having 3 columns and 4 rows).
- Based on each table, there are **THREE** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : + 3 If only the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks: – 1 In all other cases.

Answer Q. 13, Q. 14 and Q. 15 by appropriately matching the information given in the three columns of the following table.

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively.					
	Column-1		**Column-2**		**Column-3**
(I)	$x^2 + y^2 = a^2$	(i)	$my = m^2x + a$	(P)	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II)	$x^2 + a^2y^2 = a^2$	(ii)	$y = mx + a\sqrt{m^2+1}$	(Q)	$\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III)	$y^2 = 4ax$	(iii)	$y = mx + \sqrt{a^2m^2-1}$	(R)	$\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV)	$x^2 - a^2y^2 = a^2$	(iv)	$y = mx + \sqrt{a^2m^2+1}$	(S)	$\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

13. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (–1, 1), then which of the following options is the only CORRECT combination for obtaining its equation?

(a) (I) (ii) (Q) (b) (I) (i) (P) (c) (III) (i) (P) (d) (II) (ii) (Q)

Ans. (a)

Solution:

For $a = \sqrt{2}$, the equation of the circle is : $x^2 + y^2 = 2$

Equation of tangent at (–1, 1) is: $-x + y = 2$

Point of contact: $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right) \Rightarrow \left(\frac{-\sqrt{2}}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{2}}\right) \Rightarrow (-1,1)$

14. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3},\frac{1}{2}\right)$ is found to be $\sqrt{3}x+2y=4$, then which of the following options is the only CORRECT combination?

(a) (IV) (iv) (S) (b) (II) (iv) (R) (c) (IV) (iii) (S) (d) (II) (iii) (R)

Ans. (b)

Solution:

(A) $x^2+y^2=\frac{13}{4}$

Equation of tangent at $\left(\sqrt{3},\frac{1}{2}\right)$ is: $x\sqrt{3}+\frac{y}{2}=\frac{13}{4}$.

$\therefore$ option (A) is incorrect.

(B) Satisfying the point $\left(\sqrt{3},\frac{1}{2}\right)$ in the curve $x^2 + a^2y^2 = a^2$, we get $3+\frac{a^2}{4}=a^2$

$\Rightarrow \frac{3a^2}{4}=3 \Rightarrow a^2=4$ $\therefore$ the conic is: $x^2 + 4y^2 = 4$

Equation of tangent at $\left(\sqrt{3},\frac{1}{2}\right)$ is: $\sqrt{3}x+2y=4$

15. If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only CORRECT combination?

(a) (III) (i) (P) (B) (I) (ii) (Q) (c) (II) (iv) (R) (d) (III) (ii) (Q)

Ans. (a)

Solution:

The equation of given tangent is: $y = x + 8$

Satisfying the point (8, 16) in the curve $y^2 = 4ax$ we get, $a = 8$.

Now comparing the given tangent with the general tangent to the parabola, $y = mx + \frac{a}{m}$, we get $m = 1$.

Point of contact is $\left(\frac{a}{m^2},\frac{2a}{m}\right) \Rightarrow (8,16)$

Answer Q. 52, Q. 53 and Q. 54 by appropriately matching the information given in the three column of the following table.

Let $f(x) = x + \log_e x - x\log_e x$, $x\in(0,\infty)$

- Column 1 contains information about zeros of f(x), f′(x) and f″(x).
- Column 2 contains information about the limiting behaviour of f(x), f′(x) and f″(x) at infinity.
- Column 3 contains information about increasing/decreasing nature of f(x) and f′(x).

Column-1	Column-2	Column-3
(I) $f(x) = 0$ for some $x\in(1,e^2)$	(i) $\lim_{x\to\infty} f(x)=0$	(P) f is increasing in (0, 1)
(II) $f'(x) = 0$ for some $x\in(1,e)$	(ii) $\lim_{x\to\infty} f(x)=-\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x)=0$ for some $x\in(0, 1)$	(iii) $\lim_{x\to\infty} f'(x)=-\infty$	(R) f′ is increasing in (0, 1)
(IV) $f''(x) = 0$ for some $x\in(1, e)$	(iv) $\lim_{x\to\infty} f''(x)=0$	(S) f′ is decreasing in (e, e^2)

16. Which of the following options is the only INCORRECT combination?

(a) (I) (iii) (P) (b) (II) (iv) (Q) (c) (II) (iii) (P) (d) (III) (i) (R)

17. Which of the following options is the only CORRECT combination?

(a) (I) (ii) (R) (b) (III) (iv) (P) (c) (II) (iii) (S) (d) (IV) (i) (S)

18. Which of the following options is the only CORRECT combination?

(a) (III) (iii) (R) (b) (IV) (iv) (S) (c) (II) (ii) (Q) (d) (I) (i) (P)

Ans. 16. (d) 17. (c) 18. (c)

Solution:

$f(x) = x + \ell nx - x\,\ell nx$

$f'(x) = 1 + \frac{1}{x} - \ell nx - x\left(\frac{1}{x}\right) = \frac{1}{x} - \ell nx$

$f''(x) = -\frac{1}{x^2} - \frac{1}{x} < 0 \ \forall\ x \in (0, \infty)$

$\therefore$ f′(x) is strictly decreasing function for $x \in (0, \infty)$

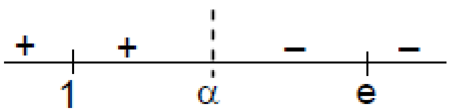
$\left.\begin{array}{l}\lim\limits_{x\to\infty} f'(x) = -\infty \\ \lim\limits_{x\to 0^+} f'(x) = \infty\end{array}\right\} \Rightarrow f'(x) = 0$ has only one real root in $(0, \infty)$

$f'(1) = 1 > 0$

$f'(e) = \frac{1}{e} - 1 < 0$

$\therefore$ f′(x) = 0 has one root in (1, e)

Let $f'(\alpha) = 0$, where $\alpha \in (1, e)$



$\therefore$ f(x) is increasing in $(0, \alpha)$ and decreasing in (α, ∞)

$f(1) = 1$ and $f(e^2) = e^2 + 2 - 2e^2 = 2 - e^2 < 0$

$\Rightarrow$ f(x) = 0 has one root in $(1, e^2)$

From column 1 : I and II are correct.

From column 2: ii, iii, and iv are correct.

From column 3: P, Q, S are correct