JEE (Mains) Mathematics Solution

26.2.2021 (Shift-1)

Section-I
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to:

(a) $\vec{a}\times\vec{b}$ (b) $\vec{0}$ (c) $\frac{1}{2}\left|\vec{a}\right|^4\vec{b}$ (d) $\left|\vec{a}\right|^4\vec{b}$

Ans. (d)
Solution:
Let $\hat{c}$ be a unit vector in the direction of $\vec{a}\times\vec{b}$.

$\Rightarrow \hat{a}\times b=\hat{c},\ b\times\hat{c}=\hat{a}\ \&\ \hat{c}\times\hat{a}=b$

$\vec{a}\times\vec{b}=\left|\vec{a}\right|\times\left|\vec{b}\right|\times\hat{c}$

$\vec{a}\times\left(\vec{a}\times\vec{b}\right)=-\left|\vec{a}\right|^2\left|\vec{b}\right|b$

$\vec{a}\times\left(\vec{a}\times\left(\hat{a}\times b\right)\right)=-\left|\vec{a}\right|^3\left|\vec{b}\right|\hat{c}$

$\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)=\left|\vec{a}\right|^4\left|\vec{b}\right|b$

$=\left|\vec{a}\right|^4\vec{b}$

2. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+3^x}dx$ is:

(a) $\frac{\pi}{4}$ (b) 2π (c) $\frac{\pi}{2}$ (d) 4π

Ans. (a)
Solution:

$$\int_{-a}^{a}f(x)dx=\int_0^a\left(f(x)+f(a-x)\right)dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+3^x}dx=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{1+3^x}+\frac{\cos^2(-x)}{1+3^{-x}}dx$$

$$=\int_0^{\frac{\pi}{2}}\cos^2 x\left(\frac{1}{1+3^x}+\frac{3^x}{1+3^x}\right)dx=\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(1+\cos 2x\right)dx=\frac{\pi}{4}$$

3. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is:

(a) 0 (b) (a + 2) (a + 3) (a + 4)

(c) – 2 (d) (a + 1) (a + 2) (a + 3)

Ans. (c)

Solution:

Given determinant is

$$D=\begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ a+5a+6 & a+3 & 1 \\ a^2+7a+12 & a+4 & 1 \end{vmatrix}$$

$R_3 \to R_3 - R_2 ; R_2 \to R_2 - R_1$

$$= \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2a+4 & 1 & 0 \\ 2a+6 & 1 & 0 \end{vmatrix}$$

Expanding by C_3

D = (2a + 4) – (2a + 6) = – 2

4. The maximum slope of the curve y = $\frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point:

(a) (2, 2) (b) (0, 0) (c) $\left(3, \frac{21}{2}\right)$ (d) (2, 9)

Ans. (a)

Solution:

y = $\frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$

Slope = y′ = $2x^3 - 15x^2 + 36x - 19$ = g(x) say

g′(x) = $6x^2 - 30x + 36$ = 6(x – 2) (x – 3)

g′(x) = 0 ⇒ $x = 2, 3$

Slope g(x) has local maximum at x = 2

x = 2 ⇒ y = 2

Local maximum at (2, 2)

[Note: Overall maximum (Absolute maximum) value of slope is far greater than that at (2, 2)].

5. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} term is equal to:

(a) 26 (b) 35 (c) 30 (d) 32

Ans. (b)

Solution:

$$a_2 + a_6 = \frac{25}{2}$$

$$a_3 \times a_5 = 25 = a_2 \times a_6 = a_4^2$$

$$a_4^2 = 25 \Rightarrow a_4 = 5$$

a_2 & a_6 are roots of $x^2 - \frac{25}{2}x + 25 = 0$

$$x = \frac{5}{2}, 10$$

$$a_2 = \frac{5}{2}, a_6 = 10 \qquad (\because \text{ GP is increasing})$$

$a_4 = 5$

$$a_4 = a_2 r^2 \Rightarrow 5 = \frac{5}{2}r^2 \Rightarrow r^2 = 2$$

$$a_8 = a_6 r^2 = 10 \times 2 = 20$$

$$a_4 + a_6 + a_8 = 5 + 10 + 20 = 35$$

6. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is:

(a) 77 (b) 42 (c) 82 (d) 35

Ans. (a)

Solution:

Combination of digits

$$3, 2, 1, 1, 1, 1, 1 \rightarrow \frac{7!}{5!} = 42$$

$$2, 2, 2, 1, 1, 1, 1 \rightarrow \frac{7!}{4!3!} = 35$$

Total = 42 + 35 = 77

7. The sum of infinite series

$1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$ is equal to:

(a) $\frac{13}{4}$ (b) $\frac{9}{4}$ (c) $\frac{11}{4}$ (d) $\frac{15}{4}$

Ans. (a)

Solution:

$$S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$$

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \frac{17}{3^5} + \ldots$$

$$\frac{2}{3}S = 1 + \frac{1}{3} + \left(\frac{5}{3^2} + \frac{5}{3^3} + \frac{5}{3^4} + \frac{5}{3^5} + \ldots\right)$$

$$= \frac{4}{3} + \frac{\frac{5}{9}}{1 - \frac{1}{3}} = \frac{5}{3} + \frac{\frac{5}{9}}{\frac{2}{3}}$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{5}{6} = \frac{13}{6}$$

$$S = \frac{13}{4}$$

8. Consider the three planes

P_1: 3x + 15y + 21z = 9,

P_2: x – 3y – z= 5, and

P_3: 2x + 10y + 14z = 5

Then, which one of the following is true?

(a) P_2 and P_3 are parallel (b) P_1 and P_3 are parallel

(c) P_1 and P_2 are parallel (d) P_1, P_2 are P_3 all are parallel

Ans. (b)

Solution:

Ratios of DRs of normals of P_1 & P_3 are $\frac{3}{2}=\frac{15}{10}=\frac{21}{14}$

$\Rightarrow$ Normals are parallel

$\Rightarrow P_1 \| P_3$

9. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is:

(a) 6 (b) 1 (c) 4 (d) 12

Ans. (c)

Solution:

Let $A=\begin{bmatrix} a & c \\ c & b \end{bmatrix}$

$$A^2=\begin{bmatrix} a & c \\ c & b \end{bmatrix}\begin{bmatrix} a & c \\ c & b \end{bmatrix}=\begin{bmatrix} a^2+c^2 & ac+bc \\ ac+bc & c^2+b^2 \end{bmatrix}$$

$a^2+b^2+2c^2=1$ as $a, b, c \in z$

c = 0 and a, b = ± 1

Total 4 matrices are possible

10. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x\in(0,1)$ is:

(a) $\frac{2.10!}{3(5!)^2}$ (b) $\frac{2.10!}{3\sqrt{3}(5!)^2}$ (c) $\frac{10!}{\sqrt{3}(5!)^2}$ (d) $\frac{10!}{2(5!)^2}$

Ans. (b)

Solution:

$$T_{r+1}={}^{10}C_r(tx^{\frac{1}{5}})^{10-r}\left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$$

For term independent of f

10 – r – r = 10 $\Rightarrow$ $r=5$

$$T_6={}^{10}C_5 x(1-x)^{\frac{1}{2}}=f(x)$$

Let

$$\therefore\ f'(x) = {}^{10}C_5\left((1-x)^{\frac{1}{2}} - \frac{x}{2(1-x)^{\frac{1}{2}}}\right) = 0$$

$$2 - 2x = x \Rightarrow x = \frac{2}{3}$$

$$f''(x) < 0 \text{ at } x = \frac{2}{3}$$

$$T_{6(\max)} = {}^{10}C_5 . \frac{2}{3}\left(\frac{1}{3}\right)^{\frac{1}{2}} = \frac{2.10!}{(5!)^2\, 3\sqrt{2}}$$

11. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is:

(a) $\frac{15}{2^{13}}$ (b) $\frac{15}{2^{12}}$ (c) $\frac{15}{2^{8}}$ (d) $\frac{15}{2^{14}}$

Ans. (a)

Solution:

Let n number of tosses

Given,

$${}^{n}C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^{n}C_9 = \left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

$$\Rightarrow n = 16$$

$\therefore$ Probability of getting 2 heads = ${}^{16}C_2\left(\frac{1}{2}\right)^{16}$

$= \frac{15}{2^{13}}$

12. The value of $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx,$ where $[x]$ is the greatest integer $\le x$, is:

(a) 100(e – 1) (b) 100(1 – e) (c) 100e (d) 100(1 + e)

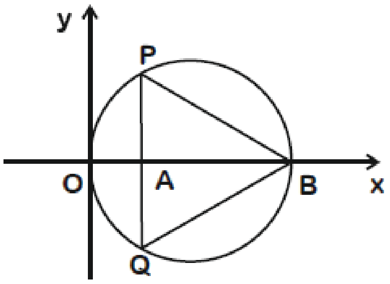
Ans. (a)

Solution:

$$\int_{n-1}^{n} e^{x-[x]}\,dx = \int_0^1 e^x dx = (e-1)$$

$$\therefore \sum_{n=1}^{100}(e-1) = 100(e-1)$$

13. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is:



(a) $24\sqrt{2}$ (b) $24\sqrt{3}$ (c) $26\sqrt{3}$ (d) $26\sqrt{2}$

Ans. (b)

Solution:

Assume that OB is diameter of the given circle

Using Ptolemy's Theorem,

OP, QB + OQ. PB = PQ × OB

$\Rightarrow\ 2OP.\,PB = 13PQ$

Also $PA^2 = OP^2 - 1 = PB^2 - 12^2$

$\Rightarrow\ PB^2 - OP^2 = 143$

and $OP^2 + PB^2 = 13^2$

then $PB^2 = 156$ and $OP^2 = 13$

$$\text{So, } PQ = \frac{2\sqrt{13}.\sqrt{156}}{13} = 4\sqrt{3}$$

$$\text{Area of } \Delta PQB = \frac{1}{2}.4\sqrt{3}.\,12 = 24\sqrt{3}$$

14. The value of $\lim\limits_{h\to 0} 2\left\{\dfrac{\sqrt{3}\sin\left(\dfrac{\pi}{6}+h\right) - \cos\left(\dfrac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cos h - \sin h\right)}\right\}$ is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{2}{\sqrt{3}}$

Ans. (a)

Solution:

$$\lim_{h\to 0} 2\left\{\frac{\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right) - \frac{1}{2}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\frac{\sqrt{3}}{2}\cos h - \frac{1}{2}\sin h\right)}\right\}$$

$$\lim_{h\to 0} 2\left\{\frac{\sin(h)}{\sqrt{3}h\left(\sin\frac{\pi}{3} - h\right)}\right\} = \frac{2}{\sqrt{3}}.\frac{2}{\sqrt{3}} = \frac{4}{3}$$

15. Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, – 1) is the set:

(a) $S = \{(x, y) | x^2 + y^2 = 2\}$ (b) $S = \{(x, y) | x^2 + y^2 = 1\}$

(c) $S = \{(x,y) | x^2 + y^2 = \sqrt{2}\}$ (d) $S = \{(x,y) | x^2 + y^2 = 4\}$

Ans. (a)

Solution:

$\therefore$ R = {(P, Q) | P and Q are at the same distance from the origin}.

Then equivalence class of (1, –1) will contain all such points which lies on circumference of the circle of centre at origin and passing through point (1, – 1).

i.e., radius of circle = $\sqrt{1^2 + 1^2} = \sqrt{2}$

$\therefore$ Required equivalence class of (S)

$= \{(x, y) | x^2 + y^2 = 2\}$.

16. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to:

(a) 16 (b) 4 (c) 8 (d) 2

Ans. (b)

Solution:

At t = 0 $B_o = 1000$

$$\frac{dB}{dt} \propto B$$

$$\Rightarrow \int_{B_o}^{1.2B_o} \frac{dB}{B} = \int_0^2 kt \qquad \text{[Given]}$$

$$\ln\left(\frac{1.2B_o}{B_o}\right) = 2k$$

$$\Rightarrow k = \frac{1}{2}\ln(1.2)$$

To find time when B = 2000

$$\Rightarrow \int_{B_o}^{2B_o} \frac{dB}{B} = \frac{1}{2}\ln(1.2)\int_0^t dt$$

$$\ln 2 = \frac{1}{2}\ln(1.2)t$$

$$\Rightarrow t = \frac{\ln 4}{\ln\left(\frac{6}{5}\right)}\text{hrs.}$$

$$\therefore R = \ln = 4$$

$$\text{Thus } \left(\frac{K}{\ln}\right)^2 = 2^2 = 4$$

17. Let f be any function defined on R and let it satisfy the condition:

$$|f(x)-f(y)| \le |(x-y)^2|,\ \forall (x, y) \in R$$

If f(0) = 1, then:

(a) f(x) can take any value in R
(b) $f(x) < 0,\ \forall x \in R$
(c) $f(x) = 0,\ \forall x \in R$
(d) $f(x) > 0,\ \forall x \in R$

Ans. (d)

Solution:

$$|f(x)-f(y)| \le |(x-y)^2|$$

$$\Rightarrow \left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$$

$$\Rightarrow \left|\lim_{x\to y}\frac{f(x)-f(y)}{x-y}\right| \le \left|\lim_{x\to y}(x-y)\right|$$

$$\Rightarrow |f'(x)| \le 0$$

$$\Rightarrow f'(x) = 0$$

$\Rightarrow$ f(x) is constant function.

$\because$ f(0) = 1 then f(x) = 1

18. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}$; $0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is:

(a) $1 - y^2$
(b) $\frac{1-y^2}{y\sqrt{y}}$
(c) $\frac{1-y^2}{1+y^2}$
(d) $\frac{1-y^2}{2y}$

Ans. (c)

Solution:

$$\because \frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c} = k \text{ (say)}$$

$\therefore \sin^{-1}x = ak$, $\cos^{-1}x = bk$ and $\tan^{-1}y = ck$

Now,

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$(a+b)x = \frac{\pi}{2}$$

$$\therefore k = \frac{\pi}{2(a+b)}$$

Now $\tan^{-1}y = \frac{\pi c}{2(a+b)}$

$$\therefore \cos\left(\frac{\pi c}{ab}\right) = \cos\left(2\tan^{-1}y\right)$$

$$= \cos\left(\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right) \quad [\text{if } y > 0]$$

$$= \frac{1-y^2}{1+y^2}$$

19. The intersection of three lines x – y = 0, x + 2y = 3 and 2x + y = 6 is a:

(a) None of the above   (b) Isosceles triangle

(c) Right angled triangle   (d) Equilateral triangle

Ans. (b)

Solution:

The given three lines are x – y = 0, x + 2y = 3 and 2x + y = 6 then point of intersection

lines x – y = 0 and x + 2y = 3 is (1, 1)

lines x – y = 0 and 2x + y = 6 is (2, 2)

and lines x +2y = 3 and 2x + y = 0 is (3, 0)

The triangle ABC has vertices A(1, 1), B(2, 2) and C(3, 0)

$\therefore\ AB=\sqrt{2}, BC=\sqrt{5}$ and $AC=\sqrt{5}$

$\therefore\ \Delta ABC$ is isosceles

20. If (1, 5, 35), (7, 5, 5), $(1, \lambda, 7)$ and $(2\lambda, 1, 2)$ are coplanar, then the sum of all possible values of λ is

(a) $\frac{44}{5}$ (b) $-\frac{44}{5}$ (c) $\frac{39}{5}$ (d) $-\frac{39}{5}$

Ans. (a)

Solution:

Four points (1, 5, 35), (7, 5, 5), $(1, \lambda, 7)$ and $(2\lambda, 1, 2)$ are coplanar then

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix}=0$$

$$\begin{vmatrix} 6 & 0 & 0 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} R_3 \to (C_3+5+C_1)=0$$

$$6((\lambda-5)(10\lambda-38)-112)=0$$

$$\therefore\ 10\lambda^2-88\lambda+78=0$$

$$\Rightarrow\ 5\lambda^2-44\lambda+39=0$$

$\therefore$ Sum of all possible values of $\lambda=\frac{44}{5}$

Section-II

Numerical Value Type Questions: This section contains 10 questions. In section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/.rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The number of integral values of 'k' for which the equation 3 sin x + 4 cos x = k + 1 has a solution, $k \in R$ is _______ .

Ans. 11

Solution:

3 sin x + 4 cos x = k + 1 has a solution then

$k+1 \in [-5, 5]$

$\therefore\ k \in [-6, 4]$

$\therefore$ Number of possible integral values of k = 11.

2. The value of the integral $\int_0^{\pi} |\sin 2x| dx$ is_______.

Ans. 2

Solution:

$$\int_0^{\pi} |\sin 2x| dx$$

$$= \int_0^{\frac{\pi}{2}} \sin 2x dx + \int_{\frac{\pi}{2}}^{\pi} -\sin 2x dx$$

$$= \left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}} + \left[\frac{\cos 2x}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= \frac{1}{2} + \frac{1}{2} + \left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

3. If $\sqrt{3}\left(\cos^2 x\right) = \left(\sqrt{3}-1\right)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is____.

Ans. 1

Solution:

$$\sqrt{3}\cos^2 x = \left(\sqrt{3}-1\right)\cos x + 1$$

$$\sqrt{3}\cos^2 x = \sqrt{3}\cos x + \cos x - 1 = 0$$

$$\sqrt{3}\cos x\left(\cos x - 1\right) + \left(\cos x - 1\right) = 0$$

$$\left(\cos x - 1\right)\left(\sqrt{3}\cos x + 1\right) = 0$$

$$\therefore\ \cos x = 1 \text{ or } -\frac{1}{\sqrt{3}}$$

$\therefore$ Number of solution in $x \in \left[0, \frac{\pi}{2}\right]$ is 1.

4. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is __________ .

Ans. 3

Solution:

$x^3 - 1 + 2x - 2x^2 = 0$

$\Rightarrow (x-1)\left[x^2-x+1\right]=0$

$\Rightarrow x=1, -\omega, -\omega^2$

$S = 1^{162} + \left(-\omega^{162}\right) + \left(-\omega^2\right)^{162}$

$= 1 + 1 + 1 = 3$

5. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x+\frac{\sqrt{a}}{2}\right)$, $a>0$ is ____ .

Ans. 2

Solution:

$y^2 = a\left(x+\frac{\sqrt{a}}{2}\right)$ ...(1)

$2y.\frac{dy}{dx} = a$ ...(2)

From (1) and (2)

$y^2 = 2y\frac{dy}{dx}\left(x+\frac{1}{2}\sqrt{2y\frac{dy}{dx}}\right)$

$y^2 - 2x\frac{dy}{dx} = y\frac{dy}{dx}.\sqrt{2y\frac{dy}{dx}}$

$\Rightarrow \left(y-2x\frac{dy}{dx}\right)^2 = 2y^3\left(\frac{dy}{dx}\right)^3$

$\Rightarrow$ Order 1 and degree 3.

6. If y = y(x) is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x$, $y(0)=0$;

then $1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to _________.

Ans. 1

Solution:

$e^{\sin y}.\cos x\frac{dy}{dx} + e^{\sin y}.\cos x = \cos x$

Let $e^{\sin y} = Y$

$\Rightarrow \frac{dY}{dx} + Y\cos x = \cos x$

$\Rightarrow$ I.F. = $e^{\sin x}$

$\Rightarrow Y.e^{\sin x} = \int e^{\sin x}.\cos\ dx + c$

$\Rightarrow e^{\sin y}.e^{\sin x} = e^{\sin x} + c$

When x = 0, y = 0 then c = 0

$\Rightarrow e^{\sin x+\sin y} = e^{\sin x} \Rightarrow {}^{\sin y} = 0$

$\Rightarrow\ y=0$

$\Rightarrow y(x)=0$

hence, $1+y\left(\frac{\pi}{6}\right)+\frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right)+\frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)=1$

7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point (4, –2, 2). If the plane is perpendicular to the line joining the points (–2, –21, 29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4$ is equal to ________ .

Ans. 8

Solution:

Vector perpendicular to the plane is

$\bar{n}=\hat{i}+5\hat{j}-6k.$

Given $A(\lambda,2,1)$ and $B(4,-2,2)$

$\because\ \overline{AB}\perp\bar{n}$, so

$(\lambda-4)+5\times4-6(1)=0$

$\Rightarrow\ \lambda-4+20+6=0$

$\Rightarrow\ \lambda=-22$

$\Rightarrow\ \frac{\lambda}{11}=-2$

hence $\left(\frac{\lambda}{11}\right)^2-4\left(\frac{\lambda}{11}\right)-4=8$

8. The number of solutions of the equation $\log_4(x-1)=\log_2(x-3)$ is ________ .

Ans. 1

Solution:

Domain: x – 1 > 0 and x – 3 > 0

$\Rightarrow\ x\in(3,\infty)$

$\because\ \log_4(x-1)=\log_2(x-3)$

$\Rightarrow\ x-1=(x-3)^2$

$\Rightarrow\ x^2-7x+8=0$

$\Rightarrow\ x=\frac{7\pm\sqrt{17}}{2}$

but only $\frac{7+\sqrt{17}}{2}$ is the correct answer.

9. Let m, n $\in$ N and gcd (2, n) = 1, if

$$30\binom{30}{0}+29\binom{30}{1}+......+2\binom{30}{28}+1\binom{30}{29}=n.2^m$$, then n + m is equal to ________. $\left(\text{Here}\binom{n}{k}={}^nC_k\right)$

Ans. 45

Solution:

$$\sum_{r=0}^{29}(30-r).\,{}^{30}C_r$$

$$=\sum_{r=1}^{30}r.{}^{30}C_{30-r}=\sum_{r=1}^{30}r.\,{}^{30}C_r$$

$$=30\sum_{r=1}^{30}{}^{29}C_{r-1}=30.2^{29}$$

$$=15.2^{30}$$

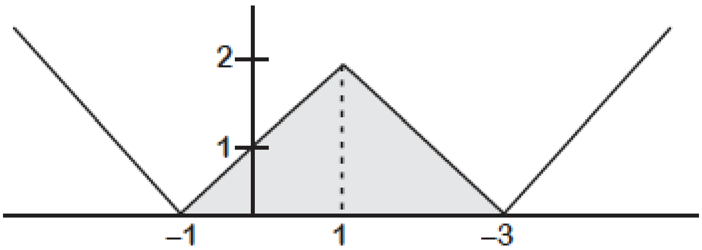
Clearly n = 15, m = 30

and m + n = 45

10. The area bounded by the lines $y=\left||x-1|-2\right|$ is _____ .

Ans. 4

Solution:



Area of the shaded region = $\frac{1}{2}(4\times 2)=4$

* As per given answer key the equation in the question should be $|y|=\left||x-1|-2\right|$