# JEE (Mains) Mathematics Question Paper

## 26.2.2021 (Shift-2)

**Section-I**

**Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

**1.** If the mirror image of the point (1, 3, 5) with respect to the plane 4x – 5y + 2z = 8 is $(\alpha, \beta, \gamma)$, then $5(\alpha+\beta+\gamma)$ equals:

(a) 43 (b) 47 (c) 41 (d) 39

**2.** A natural number has prime factorization given by $n = 2^x3^y5^z$, where y and z are such that $y + z = 5$ and $y^{-1}+z^{-1}=\frac{5}{6}, y>z.$ Then the number of odd divisors of n, including 1, is:

(a) 12 (b) 6x (c) 11 (d) 6

**3.** Let A = {1, 2, 3, ..., 10} and 10} and f: $A \to A$ be defined as

$$f(k) = \begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$$

Then the number of possible functions g: $A \to A$ such that gof = f is:

(a) $5^5$ (b) $10^5$ (c) 5! (d) ${}^{10}C_5$

**4.** Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2+y}{x}$. If the curve intersects the line x + 2y = 4 at x = – 2, then the value of y, for which the point (3, y) lies on the curve, is:

(a) $\frac{18}{35}$ (b) $-\frac{4}{3}$ (c) $-\frac{18}{11}$ (d) $-\frac{18}{19}$

**5.** The triangle of maximum area that can be inscribed in a given circle of radius ‘r’ is:

(a) An isosceles triangle with base equal to 2r.

(b) An equilateral triangle of height $\frac{2r}{3}$.

(c) A right angle triangle having two of its sides of length 2r and r.

(d) An equilateral triangle having each of its side of length $\sqrt{3}r$.

**6.** A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is;

(a) $\frac{1}{7}$ (b) $\frac{6}{7}$ (c) $\frac{4}{7}$ (d) $\frac{3}{7}$

**7.** Let $F_1(A, B, C) = (A \cap \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \to \sim A)$ be two logical expressions. Then:

(a) $F_1$ and $F_2$ both are tautologies

(b) Both $F_1$ and $F_2$ are not tautologies

(c) $F_1$ is a tautology but $F_2$ is not a tautology

(d) $F_1$ is not a tautology but $F_2$ is a tautology

**8.** Let f: $R \to R$ be defined as

$$f(x)=\begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x<-1 \\ |ax^2+x+b|, & \text{if } -1\le x\le 1 \\ \sin(\pi x), & \text{if } x>1 \end{cases}$$

If f(x) is continuous on R, then a + b equals:

(a) – 1 (b) –3 (c) 3 (d) 1

**9.** Let L be a line obtained from the intersection of two planes x + 2y + z = 6 and y + 2z = 4. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of 21 $(\alpha+\beta+\gamma)$ equals:

(a) 68 (b) 102 (c) 142 (d) 136

**10.** Let f(x) be a differentiable function at x = a with f′(a) = 2 and f(a) = 4. Then $\lim_{x\to a}\frac{xf(a)-af(x)}{x-a}$ equals:

(a) 4 – 2a (b) a + 4 (c) 2a – 4 (d) 2a + 4

**11.** Let A(1, 4) and B(1, – 5) be two points. Let P be a point on the circle $(x-1)^2+(y-1)^2=1$ such that $(PA)^2+(PB)^2$ have maximum value, then the points, P, A and B lie on:

(a) an ellipse (b) a parabola (c) a straight line (d) a hyperbola

**12.** Let $A_1$ be the area of the region bounded by the curves y = sin x, y = cos x and y-axis in the first quadrant. Also, let $A_2$ be the area of the region bounded by the curves y = sin x, y = cos x, x-axis and x = $\frac{\pi}{2}$ in the first quadrant. Then,

(a) $A_1:A_2=1:\sqrt{2}$ and $A_1+A_2=1$

(b) $A_1:A_2=1:2$ and $A_1+A_2=1$

(c) $2A_1=A_2$ and $A_1+A_2=1+\sqrt{2}$

(d) $A_1=A_2$ and $A_1+A_2=\sqrt{2}$

**13.** Let f(x) = $\int_0^x e^t f(t)dt+e^x$ be a differentiable function for all $x\in R$. Then f(x) equals:

(a) $e^{(e^x-1)}$ (b) $2e^{e^x}-1$ (c) $2e^{(e^x-1)}-1$ (d) $e^{e^x}-1$

**14.** If 0 < a, b < 1, and $\tan^{-1}a+\tan^{-1}b=\frac{\pi}{4}$, then the value of

$$(a+b)-\left(\frac{a^2+b^2}{2}\right)+\left(\frac{a^3+b^3}{3}\right)-\left(\frac{a^4+b^4}{4}\right)+\ldots \text{ is:}$$

(a) $e^2-1$ (b) $\log_e\left(\frac{e}{2}\right)$ (c) e (d) $\log_e 2$

**15.** The sum of the series $\sum_{n=1}^{\infty}\frac{n^2+6n+10}{(2n+1)!}$ is equal to:

(a) $\frac{41}{8}e+\frac{19}{8}e^{-1}-10$ (b) $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$

(c) $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$ (d) $\frac{41}{8}e+\frac{19}{8}e^{-1}+10$

**16.** For x > 0, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to:

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) – 1

**17.** If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + k$ and $\vec{a}_2 = \hat{i} + y\hat{j} + zk$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + zk$ is:

(a) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + k)$ (b) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - k)$ (c) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$ (d) $\frac{1}{\sqrt{2}}(-\hat{j} + k)$

**18.** If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to:

(a) $\frac{1}{3}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

**19.** Consider the following system of equations:

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$,

where a, b and c are real constants. Then the system of equations:

(a) has infinite number of solutions when 5a = 2b + c

(b) has a unique solution when 5a = 2b + c

(c) has a unique solution when 5a = 2b + c

(d) has a unique solution for all a, b and c

**20.** Let $f(x) = \sin^{-1}x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x\to 2} g(x)$, then the domain of the function fog is:

(a) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$ (b) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

(c) $(-\infty, -1] \cup [2, \infty)$ (d) $(-\infty, -2] \cup [-1, \infty)$

## Section-II

**Numerical Value Type Questions**: This section contains 10 questions. In section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/.rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

**1.** Let $\alpha$ and $\beta$ be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and pn+1 = 29 for some integer $n \geq 1$. Then, the value of $p_n^2$ is ________ .

**2.** The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ________.

**3.** Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is ___________ .

**4.** Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and $(4, 2\sqrt{2})$, and given that $a - 2\sqrt{2}\,b = 3$, then $(a^2 + b^2 + ab)$ is equal to________ .

**5.** Let $X_1, X_2, \ldots, X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i=\alpha)=36$ and $\sum_{i=1}^{18}(X_i=\beta)^2=90$, where $\alpha$ and $\beta$ are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha-\beta|$ is____.

**6.** If the arithmetic mean and geometric mean of the pth and qth terms of the sequence $-16, 8, -4, 2, \ldots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then p+ q is equal to _________ .

**7.** If $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, for $m, n \ge 1$, and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\alpha I_{m,n}, \alpha \in R$, then $\alpha$ equals ____.

**8.** Let z be those complex numbers which satisfy $|z+5|\le 4$ and $z(1+i)+\bar{z}(1-i)\ge -10, i=\sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is____.

**9.** If the matrix $A=\begin{bmatrix}1&0&0\\0&2&0\\3&0&-1\end{bmatrix}$ satisfies the equation $A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1&0&0\\0&4&0\\0&0&1\end{bmatrix}$ for some real numbers $\alpha$ and $\beta$, then $\beta-\alpha$ is equal to _____ .

**10.** Let a be an integer such that that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval $(a, a + 1)$. Then, $|a|$ is equal to _________ .

**ANSWER KEYS**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | b | **2.** | a | **3.** | b | **4.** | d | **5.** | d |
| **6.** | d | **7.** | d | **8.** | a | **9.** | b | **10.** | a |
| **11.** | c | **12.** | a | **13.** | c | **14.** | d | **15.** | b |
| **16.** | b | **17.** | a | **18.** | d | **19.** | a | **20.** | a |

**Integer Type**

**1. 324** **2. 1000** **3. 3** **4. 9** **5. 4**
**6. 10** **7. 1** **8. 48** **9. 4** **10. 2**