JEE (Mains) Mathematics Question Paper

26.2.2021 (Shift-1)

Section-I

Multiple Choice Questions: This section contains 20 multiple choice questions.
Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to:

(a) $\vec{a}\times\vec{b}$ (b) $\vec{0}$ (c) $\frac{1}{2}|\vec{a}|^4\vec{b}$ (d) $|\vec{a}|^4\vec{b}$

2. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2 x}{1+3^x}dx$ is:

(a) $\frac{\pi}{4}$ (b) 2π (c) $\frac{\pi}{2}$ (d) 4π

3. The value of $\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ (a+2)(a+3) & a+3 & 1\\ (a+3)(a+4) & a+4 & 1\end{vmatrix}$ is:

(a) 0 (b) (a + 2) (a + 3) (a + 4)
(c) – 2 (d) (a + 1) (a + 2) (a + 3)

4. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point:

(a) (2, 2) (b) (0, 0) (c) $\left(3, \frac{21}{2}\right)$ (d) (2, 9)

5. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4th, 6th and 8th term is equal to:

(a) 26 (b) 35 (c) 30 (d) 32

6. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is:

(a) 77 (b) 42 (c) 82 (d) 35

7. The sum of infinite series
$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+.....$ is equal to:

(a) $\frac{13}{4}$ (b) $\frac{9}{4}$ (c) $\frac{11}{4}$ (d) $\frac{15}{4}$

8. Consider the three planes
P_1: 3x + 15y + 21z = 9,
P_2: x – 3y – z= 5, and
P_3: 2x + 10y + 14z = 5
Then, which one of the following is true?

(a) P_2 and P_3 are parallel (b) P_1 and P_3 are parallel
(c) P_1 and P_2 are parallel (d) P_1, P_2 are P_3 all are parallel

9. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is:

(a) 6 (b) 1 (c) 4 (d) 12

10. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x\in(0,1)$ is:

(a) $\frac{2.10!}{3(5!)^2}$ (b) $\frac{2.10!}{3\sqrt{3}(5!)^2}$ (c) $\frac{10!}{\sqrt{3}(5!)^2}$ (d) $\frac{10!}{2(5!)^2}$

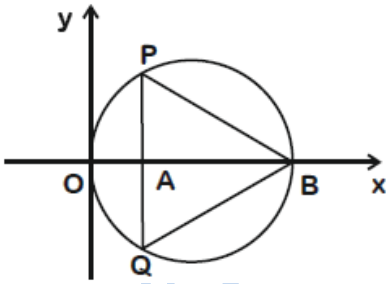
11. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is:

(a) $\frac{15}{2^{13}}$ (b) $\frac{15}{2^{12}}$ (c) $\frac{15}{2^{8}}$ (d) $\frac{15}{2^{14}}$

12. The value of $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\le x$, is:

(a) 100(e – 1) (b) 100(1 – e) (c) 100e (d) 100(1 + e)

13. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ\perp OB$. Then, the area of the triangle PQB (in square units) is:



(a) $24\sqrt{2}$ (b) $24\sqrt{3}$ (c) $26\sqrt{3}$ (d) $26\sqrt{2}$

14. The value of $\lim_{h\to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cos h-\sin h\right)}\right\}$ is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{2}{\sqrt{3}}$

15. Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, – 1) is the set:

(a) $S=\{(x,y)|\,x^2+y^2=2\}$ (b) $S=\{(x,y)|\,x^2+y^2=1\}$

(c) $S=\{(x,y)|\,x^2+y^2=\sqrt{2}\}$ (d) $S=\{(x,y)|\,x^2+y^2=4\}$

16. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to:

(a) 16 (b) 4 (c) 8 (d) 2

17. Let f be any function defined on R and let it satisfy the condition:

$|f(x)-f(y)| \le |(x-y)^2|, \ \forall (x, y) \in R$

If f(0) = 1, then:

(a) f(x) can take any value in R (b) $f(x) < 0, \ \forall x \in R$

(c) $f(x) = 0, \ \forall x \in R$ (d) $f(x) > 0, \ \forall x \in R$

18. If $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}$; $0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is:

(a) $1 - y^2$ (b) $\frac{1-y^2}{y\sqrt{y}}$ (c) $\frac{1-y^2}{1+y^2}$ (d) $\frac{1-y^2}{2y}$

19. The intersection of three lines x – y = 0, x + 2y = 3 and 2x + y = 6 is a:

(a) None of the above (b) Isosceles triangle

(c) Right angled triangle (d) Equilateral triangle

20. If (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is

(a) $\frac{44}{5}$ (b) $-\frac{44}{5}$ (c) $\frac{39}{5}$ (d) $-\frac{39}{5}$

Section-II

Numerical Value Type Questions: This section contains 10 questions. In section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/.rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The number of integral values of 'k' for which the equation 3 sin x + 4 cos x = k + 1 has a solution, $k \in R$ is _____ .

2. The value of the integral $\int_0^{\pi} |\sin 2x| dx$ is _____.

3. If $\sqrt{3}(\cos^2 x) = (\sqrt{3}-1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is ___.

4. The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is _____ .

5. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$, $a > 0$ is ___ .

6. If y = y(x) is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x, \ y(0) = 0;$

then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to _____- .

7. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point (4, –2, 2). If the plane is perpendicular to the line joining the points (–2, –21, 29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to _____ .

8. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _____ .

9. Let $m, n \in N$ and gcd (2, n) = 1, if

$30\binom{30}{0}+29\binom{30}{1}+.....+2\binom{30}{28}+1\binom{30}{29}=n.2^m$, then n + m is equal to _____ . $\left(\text{Here}\binom{n}{k}={}^nC_k\right)$

10. The area bounded by the lines $y=\left||x-1|-2\right|$ is ____ .

ANSWER KEYS

1.	D	**2.**	A	**3.**	C	**4.**	A	**5.**	B
6.	A	**7.**	A	**8.**	B	**9.**	C	**10.**	B
11.	A	**12.**	A	**13.**	B	**14.**	A	**15.**	A
16.	B	**17.**	D	**18.**	C	**19.**	B	**20.**	a

Integer Type

1. 11 **2.** 2 **3.** 1 **4.** 3 **5.** 2

6. 1 **7.** 8 **8.** 1 **9.** 45 **10.** 4