JEE (Mains) Mathematics Question Paper

24.2.2021 (Shift-2)

Section-I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. The angle of elevation of a jet plane from a point A on the ground is 60^0. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30^0. If the jet plane is flying at a constant height, then its height is:

(a) $3600\sqrt{3}$ m (b) $2400\sqrt{3}$ m (c) $1800\sqrt{3}$ m (d) $1200\sqrt{3}$ m

2. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is:

(a) $\frac{65}{2^7}$ (b) $\frac{135}{2^9}$ (c) $\frac{65}{2^8}$ (d) $\frac{35}{2^7}$

3. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are:

(a) (–2, 8) (b) (1, 5) (c) (3, 13) (d) (2, 8)

4. If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2\right)$ is:

(a) $\frac{n(2n+1)(3n+1)}{6}$ (b) $\frac{n(n-1)(2n+1)}{6}$

(c) $\frac{n(n+1)^2(n+2)}{12}$ (d) $\frac{n(n+1)(2n+1)}{6}$

5. Let f(x) be a differentiable function defined on [0, 2] such that $f'(x) = f'(2 - x)$ for all $x \in (0, 2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)dx$ is:

(a) $2(1 + e^2)$ (b) $1 + e^2$ (c) $2(1 - e^2)$ (d) $1 - e^2$

6. For the statements p and q, consider the following compound statements:

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q) \wedge \sim p) \rightarrow q$

Then which of the following statements is correct?

(a) (a) and (b) both are tautologies.

(b) (a) is a tautology but not (b).

(c) (b) is a tautology but not (a).

(d) (a) and (b) both are tautologies.

7. For the system of linear equations:

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R$, consider the following statements:

(a) The system has unique solution if $k \neq 2, k \neq -2$.

(b) The system has unique solution if k = – 2.

(c) The system has no solution if k = 2.

(d) The system has no solution if k = 2.

(e) The system has finite number of solutions if $k \neq -2$.

Which of the following statements are correct?

(a) (a) and (E) only (b) (a) and (d) only (c) (b) and (e) only (d) (c) and (d) only

8. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is:

(a) $\frac{69}{256}$ (b) $-\frac{71}{256}$ (c) $-\frac{69}{256}$ (d) $\frac{71}{256}$

9. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is:

(a) $2\sqrt{2} - 1$ (b) $\sqrt{7} - 1$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{2\sqrt{2}}$

10. Let f: $R \to R$ be defined as

$$f(x) = \begin{cases} -55x, & \text{if } x < -5 \\ 2x^3 - 3x^2 - 120x, & \text{if } -5 \le x \le 4 \\ 2x^3 - 3x^2 - 36x - 336, & \text{if } x > 4, \end{cases}$$

Let A = $\{x \in R : f \text{ is increasing}\}$. Then A is equal to:

(a) $(-5, \infty)$ (b) $(-\infty, -5) \cup (4, \infty)$

(c) $(-5, -4) \cup (4, \infty)$ (d) $(-\infty, -5) \cup (-4, \infty)$

11. The value of the integral, $\int_1^3 [x^2 - 2x - 2]dx$, where [x] denotes the greatest integer less than or equal to x, is:

(a) – 5 (b) $-\sqrt{2} - \sqrt{3} + 1$

(c) $-\sqrt{2} - \sqrt{3} - 1$ (d) – 4

12. The negation of the statement $\sim p \wedge (p \vee q)$ is:

(a) $p \wedge \sim q$ (b) $\sim p \vee q$ (c) $\sim p \wedge q$ (d) $p \vee \sim q$

13. If the curve $y = ax^2 + bx + c, x \in R$, passes through the point (1, 2) and the tangent line to this curve at origin is y = x, then the possible values of a, b, c are:

(a) $a = \frac{1}{2}, b = \frac{1}{2}, c = 1$ (b) a = 1, b = 0, c = 1

(c) a = 1, b = 1, c = 0 (d) a = – 1, b = 1, c = 1

14. For which of the following curves, the line $x+\sqrt{3y}=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(a) $x^2 + 9y^2 = 9$ (b) $y^2 = \frac{1}{6\sqrt{3}}x$ (c) $2x^2 - 18y^2 = 9$ (d) $x^2 + y^2 = 7$

15. If a curve y = f(x) passes through the point (1, 2) and satisfies $x\frac{dy}{dx}+y=bx^4$,

then for what value of b, $\int_1^2 f(x)dx=\frac{62}{5}$?

(a) $\frac{31}{5}$ (b) $\frac{62}{5}$ (c) 10 (d) 5

16. The area of the region: R = {(x, y): $5x^2 \le y \le 2x^2+9$} is:

(a) $6\sqrt{3}$ square units (b) $11\sqrt{3}$ square units

(c) $12\sqrt{3}$ square units (d) $9\sqrt{3}$ square units

17. The vector equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+k)=1$ and $\vec{r}.(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is:

(a) $\vec{r}.(\hat{i}+7\hat{j}+3k)=\frac{7}{3}$ (b) $\vec{r}.(3\hat{i}+7\hat{j}+3k)=7$

(c) $\vec{r}.(\hat{i}-7\hat{j}+3k)=\frac{7}{3}$ (d) $\vec{r}.(\hat{i}+7\hat{j}+3k)=7$

18. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has:

(a) exactly two solutions (b) infinitely many solutions

(c) no solution (d) a unique solution

19. Let $a, b\in R$. If the mirror image of the point P(a, 6, 9) with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is (20, b, –a, –9), then $|a+b|$ is equal to:

(a) 86 (b) 88 (c) 90 (d) 84

20. Let f be a twice differentiable function defined on R such that f(0) = 1, f ′(0) = 2 and f′(x) $\neq 0$ for all $x\in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0$, for all $x\in R$, then the value of f(1) lies in the interval:

(a) (0, 3) (b) (9, 12) (c) (3, 6) (d) (6, 9)

Section-II

Numerical Value Type Questions: This section contains 10 questions. In section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/.rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. The sum of first four terms of a geometric progression (G.P) is $\frac{65}{12}$ and the sum of their respectively reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is____ .

2. Let λ be an integer. If the shortest distance between the lines $x-\lambda=2y-1=-2z$ and $x=y+2\lambda=z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ________ .

3. Let $i=\sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}}+\frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}}=k$, and $n=[|k|]$ be the greatest integral part of $|k|$.

Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to _________ .

4. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is_____.

5. The students S_1, S_2, ...,S_{10} are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is _________ .

6. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (–5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ________ .

7. For integers n and r, let

$$\binom{n}{r}=\begin{cases}{}^nC_r, & \text{if } n\ge r\ge 0\\ 0, & \text{otherwise}\end{cases}$$

The maximum value of k for which the sum

$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+i-i}$ exists, is equal to _________.

8. If $a+\alpha=1, b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x},\ x\neq 0,$ then the value of the expression

$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is_____.

9. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5, 7) is A, then 24A is equal to ________ .

10. If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is________ .

ANSWER KEYS

1.	d	**2.**	b	**3.**	d	**4.**	d	**5.**	b
6.	a	**7.**	b	**8.**	b	**9.**	c	**10.**	c
11.	c	**12.**	d	**13.**	c	**14.**	a	**15.**	c
16.	c	**17.**	d	**18.**	b	**19.**	b	**20.**	d

Integer Type

1. 3 **2.** 1 **3.** 310 **4.** 2 **5.** 31650

6. 56.25* **7.** 12 **8.** 2 **9.** 1225 **10.** 11