JEE (Mains) Mathematics Question Paper

24.2.2021 (Shift-1)

Section-I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to:

(a) (−1, 3) (b) (3, 1) (c) (1, – 3) (d) (1, 3)

2. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is:

(a) $12\pi + 3\sqrt{3}$ (b) $24\pi + 3\sqrt{3}$ (c) $12\pi - 3\sqrt{3}$ (d) $24\pi - 3\sqrt{3}$

3. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + \ldots \infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is:

(a) $\frac{3}{2}$ (b) $\sqrt{3}$ (c) $2\sqrt{3}$ (d) $\frac{1}{2}$

4. The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If P(0) = 850, then the time at which population becomes zero is:

(a) $\log_e 18$ (b) $\frac{1}{2}\log_e 18$ (c) $\log_e 9$ (d) $2\log_e 18$

5. The statement among the following that is a tautology is:

(a) $A \vee (A \wedge B)$ (b) $B \to [A \wedge (A \to B)]$

(c) $[A \wedge (A \to B)] \to B$ (d) $A \wedge (A \vee B)$

6. Let p and q be two positive numbers such that p + q = 2 and $p^4 + q^4 = 272$. Then p and q are roots of the equation:

(a) $x^2 - 2x + 2 = 0$ (b) $x^2 - 2x + 8 = 0$ (c) $x^2 - 2x + 136 = 0$ (d) $x^2 - 2x + 16 = 0$

7. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if:

(a) $k \neq 3, m \neq \frac{4}{5}$ (b) $k \neq 3, m \in R$ (c) $k = 3, m = \frac{4}{5}$ (d) $k = 3, m \neq \frac{4}{5}$

8. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is:

(a) 560 (b) 1050 (c) 1625 (d) 575

9. The equation of the plane passing through the point (1, 2, – 3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:
(a) $3x - 10y - 2z + 11 = 0$
(b) $6x - y - 2z - 2 = 0$
(c) $6x - 5y + 2z + 10 = 0$
(d) $11x + y + 17z + 38 = 0$

10. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is:
(a) $-2t^3$ (b) $2t^3$ (c) 0 (d) $-t^3$

11. Let $f: R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} - R$ be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x - 1}$.
Then the composition function f(g(x)) is:
(a) neither one-one nor onto (b) onto but not one-one
(c) both one-one and onto (d) one-one but not onto

12. $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} \left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to:
(a) 0 (b) $\frac{1}{15}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

13. The ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is:
(a) $\frac{3}{16}$ (b) $\frac{1}{2}$ (c) $\frac{1}{32}$ (d) $\frac{5}{16}$

14. Two vertical poles are 150m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observed finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:
(a) $25\sqrt{3}$ (b) 30 (c) 25 (d) $20\sqrt{3}$

15. If $f: R \to R$ is a function defined by $f(x) = [x - 1] \cos\left(\dfrac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is:
(a) discontinuous only at x = 1
(b) continuous for every real x
(c) discontinuous at all integral values of x except at x = 1
(d) continuous only at x = 1

16. The distance of the point (1, 1, 9) from the point of intersection of the line $\dfrac{x-3}{1} = \dfrac{y-4}{2} = \dfrac{z-5}{2}$ and the plane $x + y + z = 17$ is:
(a) 38 (b) $19\sqrt{2}$ (c) $2\sqrt{19}$ (d) $\sqrt{38}$

17. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is
(a) x = a (b) $x = \frac{a}{2}$ (c) x = 0 (d) $x = -\frac{a}{2}$

18. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(a) C only (b) B only (c) All the three (d) A only

19. The function

$$f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x:$$

(a) Decreases in $\left[\frac{1}{2},\infty\right)$ (b) Decreases in $\left(-\infty,\frac{1}{2}\right]$

(c) Increases in $\left(-\infty,\frac{1}{2}\right]$ (d) Increases in $\left[\frac{1}{2},\infty\right)$

20. The value of

$-{}^{15}C_1 + 2.\,{}^{15}C_2 - 3.\,{}^{15}C_3 + \ldots. -15.\,{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$ is

(a) 2^{14} (b) $2^{16} - 1$ (c) $2^{13} - 13$ (d) $2^{13} - 14$

Section-II

Numerical Value Type Questions: This section contains 10 questions. In section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/.rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let A = $\{n \in N : n$ is a 3-digit number$\}$

B = $\{9k + 2 : k \in N\}$

and C = $\{9k + I : k \in N\}$ for some I $(0 < I < 9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then I is equal to ______ .

2. If $\int_{-a}^{a}(|x|+|x-2|)dx = 22, (a > 2)$ and [x] denotes the greatest integer $\leq x$, then $\int_{a}^{a}(x+[x])dx$ is equal to ________ .

3. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q=[a_{ij}]$ is a matrix satisfying PQ = KI_3 for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ________ .

4. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is _________ .

5. Let $B_i (i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occurs is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)\ p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to ______ .

6. If the least and the largest real values of α, for which the equation $z + \alpha|z-1| + 2i = 0 \left(z \in C \text{ and } i = \sqrt{-1}\right)$ has a solution, are p and q respectively, then $4\left(p^2+q^2\right)$ is equal to __________

7. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is____ .

8. Let M be any 3×3 matrix with entries from the set (0, 1, 2). The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven is _________ .

9. $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to __________- .

10. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + k$ and $\vec{b} = 2\hat{i} + k$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is ___.

ANSWER KEYS

1.	D	**2.**	D	**3.**	D	**4.**	D	**5.**	C
6.	D	**7.**	D	**8.**	C	**9.**	D	**10.**	A
11.	D	**12.**	C	**13.**	B	**14.**	A	**15.**	B
16.	D	**17.**	C	**18.**	B	**19.**	D	**20.**	d

Integer Type

1. 5 **2.** 3 **3.** 17 **4.3** **5.** 6
6. 10 **7.** 9 **8.** 540 **9.** 1 **10.** 75