JEE (Mains) Mathematics Question Paper

25.2.2021(Shift-1)

Section-I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.

1. Let α be the angle between the lines whose direction cosines satisfy the equations l + m – n = 0 and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is:

(a) $\frac{3}{4}$ (b) $\frac{5}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{8}$

2. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4,\ x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to:

(a) (5, – 8) (b) (5, 8) (c) (–5, –8) (d) (–5, 8)

3. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in:

(a) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(b) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(c) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(d) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

4. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At the point, angle of depression of the boat with the man's eye is 30^0 (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45^0. Then the time taken (in seconds) by the boat from B to reach the base of the tower is:

(a) $10\sqrt{3}$ (b) 10 (c) $10\left(\sqrt{3}+1\right)$ (d) $10\left(\sqrt{3}-1\right)$

5. The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\le t$, is:

(a) $\frac{e+1}{3}$ (b) $\frac{1}{3e}$ (c) $\frac{e+1}{3e}$ (d) $\frac{e-1}{3e}$

6. The statement $A \to (B \to A)$ is equivalent to:

(a) $A \to (A \leftrightarrow B)$ (b) $A \to (A \wedge B)$ (c) $A \to (A \to B)$ (d) $A \to (A \vee B)$

7. Let f, g: $N \to N$ such that f(n + 1) = f(n) + f(1) $\forall n \in N$ and g be any arbitrary function. Which of the following statements is NOT true?

(a) If g is onto, then fog is one-one

(b) If f is onto, then $f(n) = n \forall n \in N$

(c) f is one-one

(d) If fog is one-one, then g is one-one

8. The total number of positive integral solutions (x, y, z) such that xyz = 24 is:
(a) 36 (b) 30 (c) 45 (d) 24

9. When a missile is fired a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is:
(a) $\frac{1}{27}$ (b) $\frac{3}{8}$ (c) $\frac{3}{4}$ (d) $\frac{1}{8}$

10. Let the lines $(2-i)z=(2+i)\bar{z}$ and $(2+i)z+(i-2)\bar{z}-4i=0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz+\bar{z}+1+i=0$ is tangent to this circle C, then its radius is:
(a) $3\sqrt{2}$ (b) $\frac{3}{\sqrt{2}}$ (c) $\frac{3}{2\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

11. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90^0, then which of the following relations is TRUE?
(a) a – c = b + d (b) a + b = c + d (c) a – b = c – d (d) ab = $\frac{c+d}{a+b}$

12. The image of the point (3, 5) in the line x – y + 1 = 0, lies on:
(a) $(x-4)^2 + (y+2)^2 = 16$
(b) $(x-4)^2 + (y-4)^2 = 8$
(c) $(x-2)^2 + (y-2)^2 = 12$
(d) $(x-2)^2 + (y-4)^2 = 4$

13. The value of the integral

$$\int \frac{\sin\theta.\sin 2\theta\left(\sin^6\theta+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}\,d\theta \text{ is:}$$

(where c is a constant of integration)
(a) $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}}+c$
(b) $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$
(c) $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}}+c$
(d) $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$

14. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point:
(a) (5, 5) (b) (4, 5) (c) (4, 4) (d) (5, 4)

15. If $0<\theta,\phi<\frac{\pi}{2}$, $x=\sum_{n=0}^{\infty}\cos^{2n}\theta$, $y=\sum_{n=0}^{\infty}\sin^{2n}\phi$ and $z=\sum_{n=0}^{\infty}\cos^{2n}\theta.\sin^{2n}\phi$ then:
(a) xyz = 4 (b) xy – z = (x + y) z
(c) xy + yz + zx = z (d) xy + z = (x + y) z

16. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is:

(a) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$ (b) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$

(c) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$ (d) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$

17. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+......+\frac{1}{n}}{n^2}\right)^n$ is equal to:

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $\frac{1}{e}$

18. The coefficients a, b and c of the quadratic equation $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is:

(a) $\frac{1}{54}$ (b) $\frac{1}{36}$ (c) $\frac{5}{216}$ (d) $\frac{1}{72}$

19. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it?

(a) (0, 3) (b) (–6,0) (c) (4, 5) (d) (5, 4)

20. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is:

(a) 2 (b) 3 (c) 4 (d) 0

Section-II

Numerical Value Type Questions: This section contains 10 questions. In section II, attempt any five questions out of 10. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/.rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

1. Let A_1, A_2, A_3, ... be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is __________ .

2. The number of points, at which the function $f(x)=|2x+1|-3|x+2|+|x^2+x-2|, x\in R$ is not differentiable is_________ .

3. The total number of numbers, lying between 100 and 1000 than can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is ____________ .

4. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = – 1, and x = 1. If $\lim_{x\to0}\frac{f(x)}{x^3}=1$, then 5. f(2) is equal to _________.

5. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to __________.

6. If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to____ .

7. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}x - y - 4(\sqrt{3})k = 0$ is a conic, whose eccentricity is ____________ .

8. If the system of equations

$kx + y + 2z = 1$

$3x - y - 2z = 2$

$-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to _________ .

9. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$.

If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is ________ .

10. Let $\vec{a} = \hat{i} + 2\hat{j} - k$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - k$ be three given vectors, if $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$, then $\vec{r}.\vec{a}$ is equal to __________.

ANSWER KEYS

1.	b	**2.**	b	**3.**	d	**4.**	c	**5.**	c
6.	d	**7.**	a	**8.**	b	**9.**	d	**10.**	c
11.	c	**12.**	d	**13.**	b	**14.**	a	**15.**	d
16.	c	**17.**	c	**18.**	c	**19.**	d	**20.**	b

Integer Type

1. 9 **2.** 2 **3.** 32 **4.** 144 **5.** 64 **6.** 13

7. 2 **8.** 21 **9.** 7 **10.** 12