JEE-Main Mathematics Solution 17.3.2021 Shift-1

1. The inverse of y = $5^{\log x}$ is :

(a) $x = 5^{\log y}$ (b) $x = y^{\log 5}$ (c) $x = y^{\frac{1}{\log 5}}$ (d) $x = 5^{\frac{1}{\log y}}$

Ans. (c)

Solution:

Given y = $5^{(\log_a x)} = f(x)$

Interchanging x & y for inverse

x = $5^{(\log_a y)} = y^{(\log_a 5)}$

option (1) or option (2)

Further, from given relation

$\log_5 y = \log_a x$

$\Rightarrow x = a^{(\log_5 y)} = y^{(\log_5 a)}$

$\Rightarrow x = y^{\left(\frac{1}{\log_a 5}\right)} = f^{-1}(y)$

option (3)

2. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4k$ and $\vec{b} = 7\hat{i} + \hat{j} - 6k$.

If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r}.\left(\hat{i} + 2\hat{j} + k\right) = -3$, then $\vec{r}.\left(2\hat{i} - 3\hat{j} + k\right)$ is equal to:

(a) 12 (b) 8 (c) 13 (d) 10

Ans. (a)

Solution:

$\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\Rightarrow \vec{r} \times \left(\vec{a} - \vec{b}\right) = 0$

$\Rightarrow \vec{r} = \lambda\left(\vec{a} - \vec{b}\right)$

$\Rightarrow \vec{r} = \lambda\left(-5\hat{i} - 4\hat{j} + 10k\right)$

Also $\vec{r}.\left(\hat{i} + 2\hat{j} + k\right) = -3$

$\Rightarrow \lambda(-5 - 8 + 10) = -3$

$\lambda = 1$

Now $\vec{r} = -5\hat{i} - 4\hat{j} + 10k$

= $\vec{r}.\left(2\hat{i} - 3\hat{j} + k\right)$

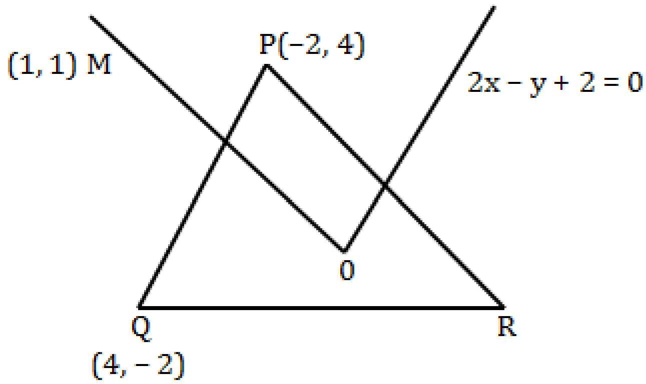
= –10 + 12 + 10 = 12

3. In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4, – 2) respectively. If the equation of the perpendicular bisector of PR is 2x – y + 2 = 0, then the centre of the circumcircle of the ΔPQR is:

(a) (–1, 0) (b) (–2, –2) (c) (0, 2) (d) (1, 4)

Ans. (b)

Solution:



Equation of perpendicular bisector of PR is y = x

Solving with 2x – y + 2 = 0 will give (–2, 2)

4. The system of equations kx + y + z = 1, x + ky + z = k and x + y + zk = k^2 has no solution if k is equal to:

(a) 0 (b) 1 (c) – 1 (d) – 2

Ans. (d)

Solution:

kx + y + z = 1

x + ky + z = k

x + y + zk = k^2

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix} = K\left(K^2-1\right)-1\left(K-1\right)+1\left(1-K\right)$$

= K^3 – K – K + 1 + 1 – K

= K^3 – 3K + 2

= $(K – 1)^2$ (K + 2)

For K = 1

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

But for K = – 2, at least one out of $\Delta_1, \Delta_2, \Delta_3$ are not zero

Hence for no solution K = – 2

5. If $\cot^{-1}(\alpha) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + \ldots$ upto 100 terms, then α is:

(a) 1. 01 (b) 1. 00 (c) 1.02 (d) 1.03

Ans. (a)

Solution:

$$\text{Cot}^{-1}(\alpha) = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18) + \ldots$$

$$= \sum_{n=1}^{1000} \tan^{-1}\left(\frac{2}{4n^2}\right)$$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$= \tan^{-1} 201 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{200}{202}\right)$

$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{202}{200}\right)$

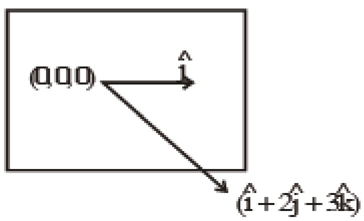
$\alpha = 1.01$

6. The equation of the plane contains the y-axis and passes through the point (1, 2, 3) is:

(a) $x + 3z = 10$ (b) $x + 3z = 0$ (c) $3x + z = 6$ (d) $3x - z = 0$

Ans. (d)

Solution:



$\vec{n} = \hat{j} \times (\hat{i} + 2\hat{j} + 3k)$

$= -3\hat{i} + 0\hat{j} + k$

So, $(-3)(x-1) + 0(y-2) + (1)(z-3) = 0$

$\Rightarrow -3x + z = 0$

7. If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Ans. (c)

Solution:

$A^2 = \sin^2\alpha I$

So, $\left|A^2 - \frac{I}{2}\right| = \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$

$\Rightarrow \sin\alpha = \pm\frac{1}{\sqrt{2}}$

8. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to:

(a) $q \Rightarrow p$ (b) $\sim q \Rightarrow p$ (c) $p \Rightarrow \sim q$ (d) $p \Rightarrow q$

Ans. (a)

Solution:

$\because\ p \to q \equiv \sim p \vee q$

So, $* \equiv v$

Thus, $p * (\sim q) \equiv p v (\sim q)$

$\equiv q \to p$

9. Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is:

(a) $\frac{4}{9}$ (b) $\frac{17}{36}$ (c) $\frac{5}{12}$ (d) $\frac{1}{2}$

Ans. (b)

Solution:

n(E) = 5 + 4 + 4 + 3 + 1 = 17

So, P(E) = $\frac{17}{36}$

10. If the fourth term in the expansion of $\left(x + x^{\log_2 x}\right)^7$ is 4480, then the value of x where $x \in N$ is equal to:

(a) 2 (b) 4 (c) 3 (d) 1

Ans. (a)

Solution:

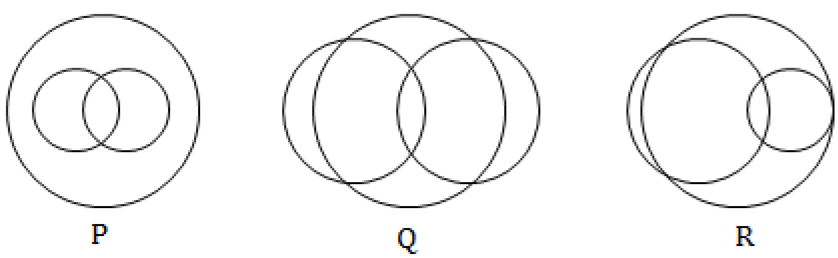
$^7C_3 x^4 x^{\left(3\log_2^x\right)} = 4480$

$\Rightarrow x^{\left(4+3\log_2^x\right)} = 2^7$

$\Rightarrow (4+3t)t = 7; t = \log_2^x$

$\Rightarrow t = 1, \frac{-7}{3} \Rightarrow x = 2$

11. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?



(a) P and Q (b) P and R (c) None of these (d) Q and R

Ans. (c)

Solution:

$A \cap B \cap C$ is visible in all three venn diagram

Hence, Option (c)

12. The sum of possible values of x for $\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\left(\frac{8}{31}\right)$ is:

(a) $-\frac{32}{4}$ (b) $-\frac{31}{4}$ (c) $-\frac{30}{4}$ (d) $-\frac{33}{4}$

Ans. (a)

Solution:

$\tan^{-1}(x+1)+\cot^{-1}\left(\frac{1}{x-1}\right)=\tan^{-1}\frac{8}{31}$

Taking tangent both sides: -

$\frac{(x+1)+(x-1)}{1-(x^2-1)}=\frac{8}{31}$

$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31}$

$\Rightarrow 4x^2+31x-8=0$

$\Rightarrow x=-8, \frac{1}{4}$

But, if $x=\frac{1}{4}$

$\tan^{-1}(x+1)\in\left(0,\frac{\pi}{2}\right)$

& $\cot^{-1}\left(\frac{1}{x-1}\right)\in\left(\frac{\pi}{2},\pi\right)$

$\Rightarrow$ LHS > $\frac{\pi}{2}$ & RHS < $\frac{\pi}{2}$

(Not possible)

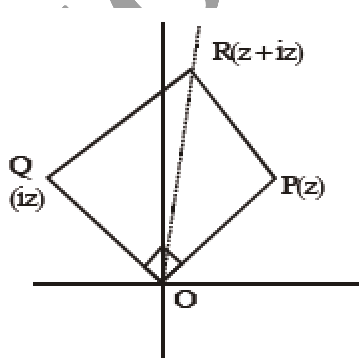
Hence, x = – 8

13. The area of the triangle with vertices A(z), B(iz) and C(z + iz) is:

(a) 1 (b) $\frac{1}{2}|z|^2$ (c) $\frac{1}{2}$ (d) $\frac{1}{2}|z+iz|^2$

Ans. (b)

Solution:



$A = \frac{1}{2}|z||iz|$

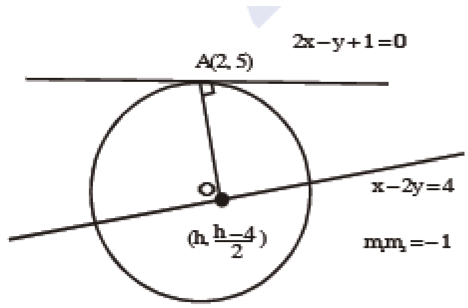
$= \frac{|z|^2}{2}$

14. The line 2x – y + 1 = 0 is a tangent to the circle at the point (2, 5) and the centre of the circle lies on x – 2y = 4. Then, the radius of the circle is:

(a) $3\sqrt{5}$ (b) $5\sqrt{3}$ (c) $5\sqrt{4}$ (d) $4\sqrt{5}$

Ans. (a)

Solution:



$$\left(\frac{h-\frac{(h-4)}{2}}{2-h}\right)(2)=-1$$

h = 8

center (8, 2)

Radius = $\sqrt{(8-2)^2+(2-5)^2} = 3\sqrt{5})$

15. Team 'A" consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to:

(a) 5 (b) 2 (c) 4 (d) 6

Ans. (c)

Solution:

Total matches between boys of both team

$= {}^7C_1 \times {}^4C_1 = 28$

Total matches between girls of both

team = ${}^nC_1\ {}^6C_1 = 6n$

Now, 28 + 6n = 52

$\Rightarrow n = 4$

16. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+......\infty}}}}$ is:

(a) $2+\frac{2}{5}\sqrt{30}$ (b) $2+\frac{4}{5}\sqrt{30}$ (c) $4+\frac{4}{5}\sqrt{30}$ (d) $5+\frac{2}{5}\sqrt{30}$

Ans. (a)

Solution:

$$y=4+\frac{1}{\left(5+\frac{1}{y}\right)}$$

$$y-4=\frac{y}{(5y+1)}$$

$$5y^2-20y-4=0$$

$$y=\frac{20+\sqrt{480}}{10}$$

$$y=\frac{20-\sqrt{480}}{10}\rightarrow \text{rejected}$$

$$y=2+\sqrt{\frac{480}{100}}$$

Correct with Option (A)

17. Choose the incorrect statement about the two circles whose equations are given below:

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(a) Distance between two centres is the average of radii of both the circles.

(b) Both circles' centres lie inside region of one another.

(c) Both circles pass through the centre of each other.

(d) Circles have two intersection points.

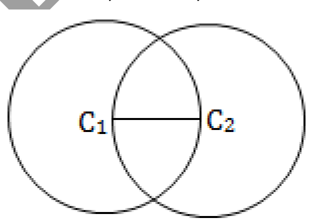
Ans. (b)

Solution:

$r_1 = 3$, $c_1 = (5, 5)$

$r_2 = 3$, c_2 $(8, 5)$

$C_1C_2 = 3$, $r_1 = 3$, $r_2 = 3$



18. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in R$ such that

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^{\alpha} x}{\cos^{\alpha} x+\sin^{\alpha} x} dx$$

(a) $g(\alpha)$ is a strictly increasing function

(b) $g(\alpha)$ has an inflection point at $\alpha=-\frac{1}{2}$

(c) $g(\alpha)$ is a strictly decreasing function

(d) $g(\alpha)$ is an even function

Ans. (d)

Solution:

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\pi/3} \frac{\sin^{\alpha} x}{(\sin^{\alpha} x+\cos^{\alpha} x)} \quad \text{...(i)}$$

$$g(\alpha)=\int_{\frac{\pi}{6}}^{\pi/3} \frac{\cos^{\alpha} x}{(\sin^{\alpha} x+\cos^{\alpha} x)} \quad \text{...(ii)}$$

(1) + (2)

$$2g(\alpha)=\frac{\pi}{6}$$

$$g(\alpha)=\frac{\pi}{12}$$

Constant and even function

19. Which of the following is true for y(x) that satisfies the differential equation

$$\frac{dy}{dx}=xy-1+x-y;\ y(0)=0:$$

(a) $y(1)=e^{-\frac{1}{2}}-1$

(b) $y(1)=e^{\frac{1}{2}}-e^{-\frac{1}{2}}$

(c) $y(1)=1$

(d) $y(1)=e^{\frac{1}{2}}-1$

Ans. (a)

Solution:

$$\frac{dy}{dx}=(1+y)(x-1)$$

$$\frac{dy}{(y+1)}=(x-1)dx$$

Integrate ln (y + 1) = $\frac{x^2}{2} - x + c$

$(0, 0) \Rightarrow c = 0 \Rightarrow y = e^{\left(\frac{x^2}{2} - x\right)} - 1$

20. The value of $\lim_{x \to 0^+} \frac{\cos^{-1}\left(x - [x]^2\right) . \sin^{-1}\left(x - [x]^2\right)}{x - x^3}$, where [x] denotes the greatest integer $\le x$ is:

(a) π (b) 0 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Ans. (d)

Solution:

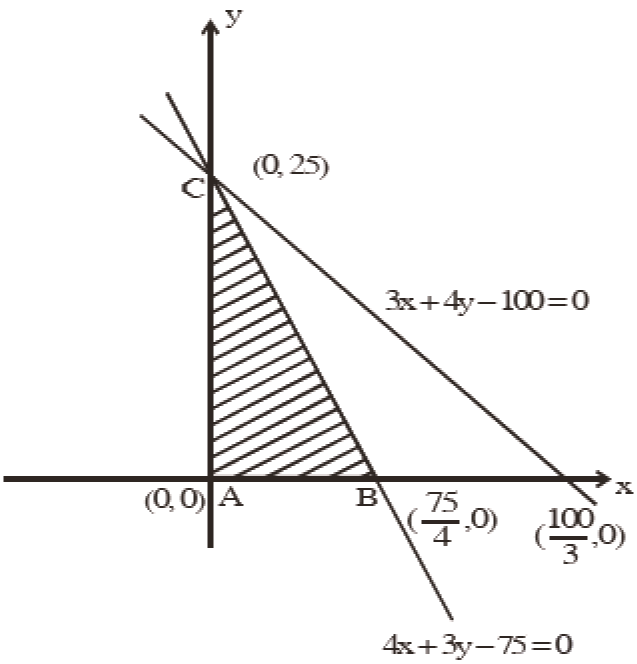
$\lim_{x \to 0^+} \frac{\cos^{-1} x}{\left(1 - x^2\right)} \times \frac{\sin^{-1} x}{x} = \frac{\pi}{2}$

Integer Type

1. The maximum value of z in the following equation z = 6xy + y^2, where 3x + 4y ≤ 100 and 4x + 3y ≤ 75 for $x \ge 0$ and $y \ge 0$ is____.

Ans. 904

Solution:



z = 6xy + y^2 = y (6x + y)

3x + 4y $\le$ 100 ...(i)

$4x + 3y \le 75$ ...(ii)

$x \ge 0$

$y \ge 0$

$x \le \frac{75 - 3y}{4}$

$Z = y(6x + y)$

$Z \le y\left(6.\left(\frac{75-3y}{4}\right)+y\right)$

$Z \le \frac{1}{2}\left(225y-7y^2\right) \le \frac{(225)^2}{2\times 4\times 7}$

$= \frac{50625}{56}$

≈ 904.0178

≈ 904.02

It will be attend at $y = \frac{225}{14}$

2. If the function $f(x) = \frac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$, then k is ________ .

Ans. 6

Solution:

$\lim_{x\to 0}\frac{\cos(\sin x)-\cos x}{x^4} = f(0)$

$\Rightarrow \lim_{x\to 0}\frac{2\sin\left(\frac{\sin x+x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4} = \frac{1}{K}$

$\Rightarrow \lim_{x\to 0}2\left(\frac{\sin x+x}{2x}\right)\left(\frac{x-\sin x}{2x^3}\right) = \frac{1}{K}$

$\Rightarrow 2\times\frac{(1+1)}{2}\times\frac{1}{2}\times\frac{1}{6} = \frac{1}{K}$

$\Rightarrow K = 6$

3. If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $\left|a^2-b^2\right|$ is ____.

Ans. 481

Solution:

$f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ at $x = 1;\ 2^{2x} = 4$

for $\sin\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$;

Let $\tan^{-1}x = \theta;\ \theta \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore\ \sin\left(\cos^{-1}\cos 2\theta\right) = \sin 2\theta$

$$\left\{\begin{array}{l}\text{If } x > 1 \Rightarrow \frac{\pi}{2} > \theta > \frac{\pi}{4} \\ \therefore \quad \pi > 2\theta > \frac{\pi}{2}\end{array}\right\}$$

$= 2 \sin\theta \cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{2x}{1+x^2}$

Hence, $f(x) = \frac{2.2^x}{1+2^{2x}}$

$\therefore \; f'(x) = \frac{(1+2^{2x})(2.2^x \ln 2) - 2^{2x}.\, 2.\ln 2.\, 2.\, 2^x}{(1+2^{2x})}$

$\therefore \; f^1(1) = \frac{20\ln 2 - 32\ln 2}{25} = -\frac{12}{25}\ln 2$

So, a = 25, b = 12 $\Rightarrow |a^2 - b^2| = 25^2 - 12^2$

= 625 –144

= 481

4. Let three be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denotes the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to ____ .

Ans. 6

Solution:

Let $P(E_1) = P_1;\ P(E_2) = P_2; P(E_3) = P_3$

$P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = \alpha = P_1(1-P_2)(1-P_3)$ ...(1)

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = \beta = (1-P_1)P_2(1-P_3)$ ...(2)

$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = \gamma = (1-P_1)P_3$ ...(3)

$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P = (1-P_1)(1-P_2)(1-P_3)$ ...(4)

Given that, $(\alpha - 2\beta)\, P = \alpha\beta$

$\Rightarrow \left(P_1(1-P_2)(1-P_3) - 2(1-P_1)P_2(1-P_3)\right) P = P_1P_2$

$(1-P_1)(1-P_2)(1-P_3)^2$

$\Rightarrow \left(P_1(1-P_2) - 2(1-P_1)P_2\right) = P_1P_2$

$\Rightarrow \left(P_1 - P_1P_2 - 2P_2 + 2P_1P_2\right) = P_1P_2$

$\Rightarrow P_1 = 2P_2$ ....(1)

and similarly, $(\beta - 3\gamma)P = 2\beta\gamma$

$P_2 = 3P_3$ ....(2)

So, $P_1 = 6P_3 \Rightarrow \boxed{\frac{P_1}{P_3} = 6}$

5. If $\vec{a} = \alpha\hat{i} + \beta\hat{j} + 3k$,

$\vec{b} = -\beta\hat{i} - \alpha\hat{j} - k$ and

$\vec{c} = \hat{i} - 2\hat{j} - k$

such that

$\vec{a}.\vec{b} = 1$ and $\vec{b}.\vec{c} = -3$, then $\frac{1}{3}\left(\left(\vec{a} \times \vec{b}\right).\vec{c}\right)$ is equal to ________ .

Ans. 2

Solution:

$\vec{a}.\vec{b} = 1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1$

$\Rightarrow -2\alpha\beta = 4 \Rightarrow \boxed{\alpha\beta = -2}$ ....(1)

$\vec{b}.\vec{c} = -3 \Rightarrow -\beta + 2\alpha + 1 = -3$

$\boxed{\beta - 2\alpha = 4}$ ....(2)

Solving (1) & (2), $(\alpha, \beta) = (-1, 2)$

$$\frac{1}{3}[\vec{a}\ \vec{b}\ \vec{c}] = \frac{1}{3}\begin{vmatrix} \alpha & \beta & 3 \\ -\beta & -\alpha & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \frac{1}{3}\begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \frac{1}{3}\begin{vmatrix} 0 & 0 & 2 \\ -2 & 1 & -1 \\ 1 & -2 & 1 \end{vmatrix} = \frac{1}{3}[2(4-1)] = 2$$

6. If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det(A^{10} - (Adj(2A))^{10})$ is equal to _____ .

Ans. 16

Solution:

$2A\ adj\ (2A) = |2A|\ I$

$\Rightarrow A\ adj\ (2A) = -4I$ ...(i)

Now, $E = |A^4| + \left|A^{10} - (adj\ (2A))^{10}\right|$

$$= (-2)^4 + \frac{\left|A^{20} - A^{10}(\text{adj } 2A)^{10}\right|}{|A|^{10}}$$

$$= 16 + \frac{\left|A^{20} - (A \text{ adj}(2A))^{10}\right|}{|A|^{10}}$$

$$= 16 + \frac{\left|A^{20} - 2^{10} I\right|}{2^{10}} \qquad (\text{from}(1))$$

Now, characteristic roots of A are 2 and –1.

So, characteristic roots of A^{20} are 2^{10} and 1.

Hence, $(A^{20} - 2^{10} I)(A^{20} - I) = 0$

$\Rightarrow \left|A^{20} - 2^{10} I\right| = 0 \ (\text{as } A^{20} \neq I)$

$\Rightarrow E = 16$ Ans.

7. If [.] represents the greatest integer function, then the value of

$$\left|\int_0^{\sqrt{\frac{\pi}{2}}} \left[\left[x^3\right] - \cos x\right] dx\right| \text{ is \_\_\_\_\_ .}$$

Ans. 1

Solution:

$$I = \int_0^{\sqrt{\pi/2}} \left(\left[x^2\right] + \left[-\cos x\right]\right) dx$$

$$= \int_0^1 0 \ dx + \int_1^{\sqrt{\pi/2}} dx + \int_0^{\sqrt{\pi/2}} (-1)\, dx$$

$$= \sqrt{\frac{\pi}{2}} - 1 - \sqrt{\frac{\pi}{2}} = -1$$

$$\Rightarrow |I| = 1$$

8. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 24x - 10y + 160 = 0$ is ___________ .

Ans. 1

Solution:

Given $C_1(5, 5)$, $r_1 = 3$ and C_2 $(12, 5)$, $r_2 = 3$

Now, $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)_{min} = 7 - 6 = 1$

Diagram pg 8 Que 8

9. If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz - 7 = 0$, then the value of $2a + b + c - 7$ is __________ .

Ans. 4

Solution:

Required plane is

$p_1 + \lambda p_2 = (2+3\lambda)x - (7+5\lambda)y + (4+4\lambda)z - 3 + 11\lambda = 0;$

which is satisfied by (–2, 1, 3).

Hence, $\lambda = \frac{1}{6}$

Thus, plane is 15x – 47y + 28z – 7 = 0

So, 2a + b + c – 7 = 4

10. If $(2021)^{3762}$ is divided by 17, then the remainder is __________ .

Ans. 4

Solution:

$(2023-2)^{3762} = 2023k_1 + 2^{3762}$

$= 17k_2 + 2^{3762}$ (as $2023 = 17 \times 17 \times 9$)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17-1)^{940}$

$= 17k_2 + 4(17k_3 + 1)$

$= 17k + 4 \Rightarrow$ remainder $= 4$