JEE-Main Mathematics Questions Paper 18.3.2021 Shift-2

1. Let y = y(x) be the solution of the differential equation

$\frac{dy}{dx}=(y+1)\left((y+1)e^{x^2/2}-x\right), 0<x<2.1,\ y(2)=0$. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to:

(a) $\frac{-e^{3/2}}{(e^2+1)^2}$ (b) $-\frac{2e^2}{(1+e^2)^2}$ (c) $\frac{e^{5/2}}{(1+e^2)^2}$ (d) $\frac{5e^{1/2}}{(e^2+1)^2}$

2. In a triangle ABC, if $|\overrightarrow{BC}|=8, |\overrightarrow{CA}|=7, |\overrightarrow{AB}|=10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to:

(a) $\frac{25}{4}$ (b) $\frac{85}{14}$ (c) $\frac{127}{20}$ (d) $\frac{115}{16}$

3. Let the system of linear equations

$4x+\lambda y+2z=0$

$2x-y+z=0$

$\mu x+2y+3z=0, \lambda, \mu \in R.$

has a non-trivial solution. Then which of the following is true?

(a) $\mu=6, \lambda \in R$

(b) $\lambda=2, \mu \in R$

(c) $\lambda=3, \mu \in R$

(d) $\mu=-6, \lambda \in R$

4. Let f: $R-\{3\} \to R-\{1\}$ be defined by f(x) = $\frac{x-2}{x-3}$. Let g: $R \to R$ be given as g(x) = 2x – 3.

Then, the sum of all the values of x for which $f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$ is equal to

(a) 7 (b) 2 (c) 5 (d) 3

5. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to:

(a) $\frac{9}{\sqrt{2}}$ (b) $7\sqrt{2}$ (c) $2\sqrt{2}$ (d) $3\sqrt{2}$

6. Consider a hyperbola H: $x^2-2y^2=4$. Let the tangent at a point $P(4,\sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1>0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(a) $4\sqrt{6}$ (b) $\sqrt{6}-1$ (c) $\frac{7}{\sqrt{6}}-2$ (d) $4\sqrt{6}-1$

7. If P and Q are two statements, then which of the following compound statement is a tautology?

(a) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

(b) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

(c) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$

(d) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

8. Let $g(x) = \int_0^x f(t)dt$, where f is continuous function in [0, 3] such that $\frac{1}{3} \le f(t) \le 1$ for all $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which g(3) lies is:

(a) $\left[-1, -\frac{1}{2}\right]$ (b) $\left[-\frac{3}{2}, -1\right]$ (c) $\left[\frac{1}{3}, 2\right]$ (d) [1, 3]

9. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to:

(a) 1000 (b) 7000 (c) 5000 (d) 3000

10. Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to:

(a) 4 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2

11. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to:

(a) 425 (b) 650 (c) 250 (d) 925

12. Let S_1: $x_2 + y_2 = 9$ and S_2: $(x - 2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points:

(a) $(0, \pm\sqrt{3})$ (b) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$ (c) $\left(2, \pm\frac{3}{2}\right)$ (d) $(1, \pm 2)$

13. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a} + \vec{b} + (\vec{a} \times \vec{b})\right)$ and $\vec{a}$ is equal to:

(a) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (c) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

14. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to:

(a) $\frac{32}{625}$ (b) $\frac{80}{243}$ (c) $\frac{40}{213}$ (d) $\frac{128}{625}$

15. Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to:

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

16. Define a relation R over a class of $n \times n$ real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true?
(a) R is symmetric, transitive but not reflexive,
(b) R is reflexive, symmetric but not transitive
(c) R is an equivalence relation
(d) R is reflexive, transitive but not symmetric

17. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to:
(a) $\frac{2\sqrt{3}}{3}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

18. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to:
(a) 350   (b) 500   (c) 400   (d) 250

19. The area bounded by the curve
$4y^2 = x^2(4 - x)(x - 2)$ is equal to:
(a) $\frac{\pi}{8}$ (b) $\frac{3\pi}{8}$ (c) $\frac{3\pi}{2}$ (d) $\frac{\pi}{16}$

20. Let f: $R \to R$ be a function defined as $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x}, & \text{if } x < 0 \\ b, & \text{if } x = 0 \\ \frac{\sqrt{x + bx^3} - \sqrt{x}}{bx^{5/2}}, & \text{if } x > 0 \end{cases}$

If f is continuous at x = 0, then the value of a + b is equal to:
(a) $-\frac{5}{2}$ (b) – 2   (c) – 3   (d) $-\frac{3}{2}$

Integer Type

1. If f(x) and g(x) are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then P(1) is equal to ________ .

2. Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to ________ .

3. If $\sum_{r=1}^{10} r!\left(r^3 + 6r^2 + 2r + 5\right) = \alpha(11!)$, then the value of α is equal to ________ .

4. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to ______________ .

5. Let P(x) be a real polynomial of degree 3 which vanishes at x = – 3. Let P(x) have local minima at x = 1, local maxima at x = – 1 and $\int_{-1}^{1} P(x)dx = 18$, then the sum of all the coefficients of the polynomial P(x) is equal to ________ .

6. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) - b = 0$ be (–3, 5, 2). Then the value of $|a+b|$ is equal to ____ .

7. Let f: $R \to R$ satisfy the equation f(x + y) = f(x). f(y) for all x, $y \in R$ and $f(x) \neq 0$ for any $x \in R$. If the function f is differentiable at x = 0 and f'(0) = – 3, then $\lim_{h \to 0} \frac{1}{h}(f(h) - 1)$ is equal to____ .

8. Let ^nC_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.
If $\sum_{k=0}^{10} (2^2 + 3k)\, ^nCk = \alpha.3^{10} + \beta.2^{10}$, $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to ________ .

9. Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to _________ .

10. Let y = y(x) be the solution of the differential equation xdy – ydx = $\sqrt{(x^2 - y^2)}$ dx, $x \geq 1$, with y(1) = 0. If the area bounded by the line x = 1, x = e^{π}, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to __________ .

ANSWER KEYS

Single Correct Type

1.	a	**2.**	b	**3.**	a	**4.**	c	**5.**	a
6.	c	**7.**	b	**8.**	c	**9.**	d	**10.**	b
11.	a	**12.**	c	**13.**	b	**14.**	a	**15.**	c
16.	c	**17.**	b	**18.**	d	**19.**	c	**20.**	d

Integer Type

1. 0	**2.** 6	**3. 160**	**4. 210**	**5. 8**
6. 1	**7. 3**	**8. 19**	**9.** 38	**10. 4**