JEE-Main Mathematics Questions Paper 17.3.2021 Shift-2

1. Let f: $R \to R$ be defined as f(x) = e^{-x} sin x. If F: [0, 1] $\to R$ is a differentiable function such that F(x) = $\int_0^x f(t)dt$, then the value of $\int_0^1 (F'(x)+f(x))e^x dx$ lies in the interval

(a) $\left[\frac{327}{360}, \frac{329}{360}\right]$ (b) $\left[\frac{330}{360}, \frac{331}{360}\right]$ (c) $\left[\frac{331}{360}, \frac{334}{360}\right]$ (d) $\left[\frac{335}{360}, \frac{336}{360}\right]$

2. If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{x-[x]}} dx = \alpha e^{-1} + \beta e^{-\frac{1}{2}} + \gamma$, where α, β, γ are integers and [x] denotes the greatest integer less than or equal to x, then the value of $\alpha+\beta+\gamma$ is equal to:

(a) 0 (b) 20 (c) 25 (d) 10

3. Let y = y(x) be the solution of the differential equation cos x (3 sin x + cos x + 3) dy = (1 + y sin x (3 sin x + cos x + 3) dx, $0 \le x \le \frac{\pi}{2}, y(0) = 0$. Then, $y\left(\frac{\pi}{3}\right)$ is equal to:

(a) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (b) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$ (c) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$ (d) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

4. The value of $\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to:

(a) 1124 (b) 1324 (c) 1024 (d) 924

5. The value of $\lim_{n\to\infty}\frac{[r]+[2r]+.....+[nr]}{n^2}$, where r is non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to:

(a) $\frac{r}{2}$ (b) r (c) 2r (d) 0

6. The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$, for $x \in [-1, 1]$, and [x] denotes the greatest integer less than or equal to x, is:

(a) 2 (b) 0 (c) 4 (d) Infinite

7. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to:

(a) $\frac{1}{18}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$

8. The number of solutions of the equation x + 2 tan x = $\frac{\pi}{2}$ in the interval $[0, 2\pi]$ is:

(a) 3 (b) 4 (c) 2 (d) 5

9. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in C : |z-1| \le \sqrt{2}\}$

$S_2 = \{z \in C : \text{Re}((1-i)z) \ge 1\}$

$S_3 = \{z \in C : \text{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$

(a) is a singleton

(b) has exactly two elements

(c) has infinitely many elements

(d) has exactly three elements

10. If the curve y = y(x) is the solution of the differential equation

$2(x^2 + x^{5/4})dy - y(x + x^{1/4})(dx = 2x^{9/4}$, $x > 0$ which passes through the point $\left(1, 1-\frac{4}{3}\log_e 2\right)$,

then the value of y(16) is equal to:

(a) $4\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$ (b) $\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$ (c) $4\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$ (d) $\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

11. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to:

(a) 364 (b) 240 (c) 333 (d) 360

12. If x, y, z are in arithmetic progression with common difference d, $x \ne 3d$, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is

(a) 72 (b) 12 (c) 36 (d) 6

13. Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{j} - k$ and $\overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3xk$, $x, y \in R$, $x > 0$, be such that $|\overrightarrow{PQ}| = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7k, z \in R$,

is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to

(a) 7 (b) 9 (c) 2 (d) 1

14. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is:

(a) 11 : 4 (b) 9 : 4 (c) 3 : 1 (d) 2 : 1

15. Consider the function $f : R \to R$ defined by $f(x) = \begin{cases} \left(2 - \sin\left(\frac{1}{x}\right)\right)|x|, & x \ne 0 \\ 0, & x = 0 \end{cases}$. Then f is:

(a) monotonic on $(-\infty, 0) \cup (0, \infty)$

(b) not monotonic on $(-\infty, 0)$ and $(0, \infty)$

(c) monotonic on $(0, \infty)$ only

(d) monotonic on $(-\infty, 0)$ only

16. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{b}=1$, then the value of b is equal to:

(a) 11 (b) 14 (c) 16 (d) 20

17. The value of the limit $\lim_{\theta\to 0}\frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$ is equal to:

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$ (c) 0 (d) $\frac{1}{4}$

18. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3, 4) meet x-axis and y-axis at point P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to

(a) $\frac{529}{64}$ (b) $\frac{125}{72}$ (c) $\frac{625}{72}$ (d) $\frac{585}{66}$

19. If the Boolean expression $(p\wedge q)\odot(p\otimes q)$ is tautology, then $\odot$ and $\otimes$ are respectively given by

(a) $\to, \to$ (b) $\wedge, \vee$ (c) $\vee, \to$ (d) $\wedge, \to$

20. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$ and containing the line $\frac{x-2}{3}=\frac{1-y}{2}=\frac{z+1}{1}$ is $\alpha x+\beta\gamma+\gamma z=24$, then $\alpha+\beta+\gamma$ is equal to:

(a) 20 (b) 19 (c) 18 (d) 21

Integer Type

1. If 1, $\log_{10}(4x - 2)$ and $\log_{10}\left(4^x+\frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to:

2. Let f: $[-1, 1] \to R$ be defined as $f(x) = ax^2 + bx + c$ for all $x\in[-1, 1]$, where a, b, c $\in R$ such that $f(-1) = 2$, $f'(-1) = 1$ and for $x\in(-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \le\alpha$, $x\in[-1, 1]$, then the least value of α is equal to ________ .

3. Let f: $[-3, 1] \to R$ be given as $f(x) = \begin{cases}\min\{(x+6), x^2\}, & -3\le x\le 0\\ \max\{\sqrt{x}, x^2\}, & 0\le x\le 1.\end{cases}$

If the area bounded by y = f(x) and x-axis is A, then the value of 6A is equal to ________ .

4. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to:

5. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ________ .

6. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12: 8: 3. Then the term independent of x in the expansion, is equal to _________ .

7. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to ___________ .

8. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to_____ .

9. Let $I_n = \int_1^e x^{19} (\log|x|)^n \, dx$, where $n \in N$. If $(20) I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equal to _________ .

10. Let P be an arbitrary point having sum of the squares of the distance from the planes x + y + z = 0, lx – nz = 0 and x – 2y + z = 0, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of l – n is equal to __________ .

ANSWER KEYS

Single Correct Type

1.	b	**2.**	a	**3.**	b	**4.**	d	**5.**	a
6.	b	**7.**	d	**8.**	a	**9.**	c	**10.**	c
11.	c	**12.**	a	**13.**	b	**14.**	b	**15.**	b
16.	b	**17.**	a	**18.**	c	**19.**	a	**20.**	b

Integer Type

1. 2 **2.** 5 **3.** 41 **4.** 144 **5.** 68

6. 4 **7.** 2020 **8.** 486 **9.** 1 **10.** 0