JEE-Main Mathematics Questions Paper 16.3.2021 Shift-2

1. The maximum value of

$$f(x)=\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x\in R \text{ is:}$$

(a) $\sqrt{7}$ (b) $\frac{3}{4}$ (c) $\sqrt{5}$ (d) 5

2. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to:

(a) $\frac{9}{56}$ (b) $\frac{4}{9}$ (c) $\frac{3}{7}$ (d) $\frac{11}{27}$

3. Let $\alpha \in R$ be such that the function

$$f(x)=\begin{cases} \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \\ \alpha, & x=0 \end{cases}$$

is continuous at x = 0, where {x} = x – [x], [x] is the greatest integer less than or equal to x. Then:

(a) $\alpha=\frac{\pi}{\sqrt{2}}$ (b) $\alpha=0$ (c) no such α exists (d) $\alpha=\frac{\pi}{4}$

4. If (x, y, z) be an arbitrary point lying on a plane P which passes through the point (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

(a) 0 (b) 3 (c) 39 (d) – 45

5. Consider the integral $I=\int_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx,$

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to:

(a) 9(e – 1) (b) 45(e + 1) (c) 45(e – 1) (d) 9(e + 1)

6. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line y = x. Then the equation of tangent to C at P(2, 1) is:

(a) x – y = 1 (b) 2x + y = 5 (c) x + 3y = 5 (d) x + 2y = 4

7. If y = y(x) is the solution of the differential equation $\frac{dy}{dx}+(\tan x)y=\sin x, 0\le x\le\frac{\pi}{3}$, with y(0) = 0, then $y\left(\frac{\pi}{4}\right)$ equal to:

(a) $\frac{1}{4}\log_e 2$ (b) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$ (c) $\log_e 2$ (d) $\frac{1}{2}\log_e 2$

8. Let A = {2, 3, 4, 5,..., 30} and '$\simeq$' be an equivalence relation on $A\times A$, defined by $(a, b)\simeq(c, d)$, if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to:

(a) 5 (b) 6 (c) 8 (d) 7

9. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax = 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to:

(a) $\sqrt{11}$ (b) $\sqrt{7}$ (c) $\sqrt{6}$ (d) $\sqrt{10}$

10. The least value of $|z|$ where z is complex number which satisfy the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|,\ i=\sqrt{-1}, \text{ is equal to:}$$

(a) 3 (b) $\sqrt{5}$ (c) 2 (d) 8

11. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to:

(a) 795 (b) 1173 (c) 1890 (d) 717

12. If the point of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to:

(a) 12 (b) 5 (c) 6 (d) 10

13. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1} x$ is equal to:

(a) 2 (b) 1 (c) 3 (d) 0

14. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A^2$, then the value of m is equal to:

(a) $\frac{4}{15}$ (b) 1 (c) 2 (d) 3

15. Let f be a real valued function, defined on R – {–1, 1} and given by

$$f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function f(x) is increasing?

(a) $(-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty\right) - \{1\}\right)$

(b) $(-\infty, \infty) - \{-1, 1\}$

(c) $\left(-1, \frac{1}{2}\right]$

(d) $\left(-\infty, \frac{1}{2}\right] - \{-1\}$

16. Let f: $S \to S$ where S = $(0, \infty)$ be a twice differentiable function such that f(x + 1) = xf(x).
If g: $S \to R$ be defined as g(x) = $\log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to:

(a) $\frac{205}{144}$ (b) $\frac{197}{144}$ (c) $\frac{187}{144}$ (d) 1

17. Let P(x) = $x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by (x – 2). Then the value of 9(b + c) is equal to:

(a) 9 (b) 15 (c) 7 (d) 11

18. If the foot of the perpendicular from point (4, 3, 8) on the line L_1: $\frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line L_2: $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to:

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{6}}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\frac{1}{\sqrt{3}}$

19. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx} = y^2 - x^2$, $x > 0$.

Let the curve C_2 be the solution of $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curve C_1 and C_2 is equal to:

(a) $\pi - 1$ (b) $\frac{\pi}{2} - 1$ (c) $\pi + 1$ (d) $\frac{\pi}{4} + 1$

20. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r}.(\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$ and $\vec{r}.(2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$, $\alpha \in R$, then the value of $\alpha + |\vec{r}|^2$ is equal to:

(a) 9 (b) 15 (c) 13 (d) 11

Integer Type

1. If the distance of the point (1, –2, 3) from the plane x + 2y – 3z + 10 = 0 measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to _______.

2. Consider the statistics of two sets of observations as follows:

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _________ .

3. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries such that A = XB, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2 + 1)b_2^2 \neq -2b_1b_2$, then the value of k is _____ .

4. For real numbers α, β, γ and δ, if $\int \frac{\left(x^2-1\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{\left(x^4+3x^2+1\right)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$

$= \alpha \log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\beta\tan^{-1}\left(\frac{\gamma\left(x^2-1\right)}{x}\right)+\delta\tan^{-1}\left(\frac{x^2+1}{x+C}\right)$

where C is an arbitrary constant, then the value of $10\left(\alpha+\beta\gamma+\delta\right)$ is equal to ________ .

5. Let f: $R \to R$ and $g: R \to R$ be defined as $f(x)=\begin{cases}x+a, & x<0\\ |x-1|, & x\geq 0\end{cases}$ and $g(x)=\begin{cases}x+1, & x<0\\ (x-1)^2+b, & x\geq 0\end{cases}$

where a, b are non-negative real numbers. If (gof) (x) is continuous for all $x \in R$, then a + b is equal to ________ .

6. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P., where a, b > 0. Then 72(a + b) is equal to ________ .

7. In ΔABC, the lengths of sides AC and AB are 12cm and 5cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to __________ .

8. Let n be a positive integer. Let $A=\sum_{k=0}^{n}(-1)^k\, {}^nC_k\left[\left(\frac{1}{2}\right)^k+\left(\frac{3}{4}\right)^k+\left(\frac{7}{8}\right)^k+\left(\frac{15}{16}\right)^k+\left(\frac{31}{32}\right)^k\right]$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to __________ .

9. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-k$ and $\vec{b}=\hat{i}+2\hat{j}+k$.

If $\vec{c}.\left(\hat{i}+\hat{j}+3k\right)=8$ then the value of $\vec{c}.\left(\vec{a}\times\vec{b}\right)$ is equal to _________ .

10. Let $S_n(x)=\log_{a^{1/2}}x+\log_{a^{1/3}}x+\log_{a^{1/6}}x+\log_{a^{1/11}}x+\log_{a^{1/18}}x+\log_{a^{1/27}}x+\ldots$ up to n-terms, where a > 1.
If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to ________ .

ANSWER KEYS

Single Correct Type

1.	c	**2.**	b	**3.**	c	**4.**	b	**5.**	c
6.	a	**7.**	b	**8.**	d	**9.**	c	**10.**	a
11.	d	**12.**	a	**13.**	c	**14.**	b	**15.**	a
16.	a	**17.**	c	**18.**	b	**19.**	b	**20.**	b

Integer Type

1. 2 **2.** 5 **3.** 1 **4.** 6 **5.** 1
6. 14 **7.** 15 **8.** 6 **9.** 28 **10.** 16