Polynomial

Chapter Flowchart

The Chapter Flowcharts give you the gist of the chapter flow in a single glance.

Polynomial

A polynomial p(x) in one variable x is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} .... + a_2x^2 + a_1x + a_0$, where a_0, a_1, a_2, ….a_n are constants and $a_n \neq 0$ and n is a positive integer, a_0, a_1, a_2, ….a_{n-1}, a_n, are respectively the coefficients of x^0, x, x^2, …x^{n-1}, x^n Each of $a_n x^n$, $a_{n-1}x^{n-1}$, $a_{n-2}x^{n-2}$ ……., a_0, with $a_n \neq 0$ is called a term of the polynomial p(x).

- **Degree of polynomial**: The highest power of variable x in a polynomial p(x) is called the degree of the polynomial p(x). For example, the degree of polynomial $2x^6 – 2x^5 + x + 3$ is 6.

On the basis of number of terms

- **Monomial:** A polynomial having one term is called a monomial 2x, $5y^2$, $7x^3$, $3x^2y$ and 2xyz are same examples of monomials.
- **Binomial**: A polynomial having two terms is called a binomial. x + 2, $x^2 – 9$, x + y, $x^2 – y^2$ are some examples of binomials.
- **Trinomial**: A polynomial having three terms is called a trinomial. For example $x^2 + 2xy + y^2$, x + y + z, etc. are trinomials.

On the basis of degree

- **Constant polynomial:** A polynomial of degree zero is called constant polynomial. For example p(x) = 3, g(x) = –7, f(x) = $\frac{3}{4}$, etc. are constant polynomials.
- **Linear polynomial**: A polynomial of degree one is called a linear polynomial, For example, p(x) = x + 3, g(x) = 2x – 5, f(x) = –3x + 7, etc. are linear polynomials. In general, p(x) = ax + b, a $\neq$ 0 is a linear polynomials.
- **Quadratic polynomial**: A polynomial of degree two is called a quadratic polynomial. For example p(x) = $x^2 + 6x + 9$, q(x) = $4y^2 – 20y + 25$, etc. are quadratic polynomials. In general p(x) = $ax^2 + bx + c$, a $\neq$ 0 is a quadratic polynomial.
- **Cubic polynomial:** A polynomial of degree three is called a cubic polynomial. For example p(x) = $x^3 + 9x^2 + 27x + 27$, q(y) = $y^3 – 6y^2 + 12y – 8$, are cubic polynomials. In general p(x) = ax3 + $bx^2 + cx + d$, $a \neq 0$ is a cubic polynomial.

Zero of a polynomial : A real number 'a' is zero of a polynomial p(x) if p(a) = 0. Here 'a' is called a root of the equation p(x) = 0.
Every linear polynomial in one variable has a unique zero.

- A non-zero constant polynomial has no zero and every real number is zero of the zero polynomial.

Remainder theorem : If p(x) is any polynomial of degree greater than or equal to 1 and p(x) is divided by the linear polynomial x – a, then the remainder is p(a).

Factor theorem : If p(x) is a polynomial of degree $n \geq 1$ and a is any real number.
Then (i) (x – a) is a factor of p(x), if p(a) = 0 and (ii) p(a) = 0, if (x – a) is a factor of p(x).

Some Algebraic Identities :

(i) $(x + y)^2 = x^2 + 2xy + y^2$

(ii) $(x – y)^2 = x^2 – 2xy + y^2$

(iii) $(x^2 – y^2) = (x + y) (x – y)$

(iv) $(x + a) (x + b) = x^2 + (a + b) x + ab$

(v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)= x^3 + 3x^2y + 3xy^2+ y^3$

(vii) $(x – y)^3 = x^3 – y^3 – 3xy (x – y) = x^3 – 3x^2y + 3xy^2 – y^3$

(viii) $x^3 + y^3 + z^3 – 3xyz = (x + y + z) (x^2 + y^2 + z^2 – xy – yz – zx)$

If x + y + z = 0, then $x^3 + y^3 + z^3 = 3xyz$

(ix) $x^3 + y^3 = (x + y) (x^2 – xy + y^2)$

(x) $x^3 – y^3 = (x – y) (x^2 + xy + y^2)$

Revision Question Bank

1. Find the remainder when $4x^3 - 3x^2 + 2x - 4$ is divided by

(a) $x - 1$ (b) $x + 2$ (c) $x + \frac{1}{2}$

2. Find the value of k, if $(x - 2)$ is a factor of the polynomial $p(x) = kx^2 - \sqrt{2}x + 1$.

3. Evaluate $(103)^3$ by using suitable identities.

4. Factorise: $3u^3 - 4u^2 - 12u + 16$.

5. Expand each of the following

(a) $(-x + 2y - 3z)^2$ (b) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$

6. For what value of m is $2x^3 + mx^2 + 11x + m + 3$ exactly divisible by $(2x - 1)$?

7. If $x + \frac{1}{x} = 7$, then find the value of $x^3 + \frac{1}{x^3}$.

8. Factorise: $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

9. The polynomial $p(x) = 4x^3 - 2x^2 + px + 5$ and $q(x) = x^3 + 6x^2 + p$, leave the remainders a and b respectively, when divided by $(x+2)$. Find the value of p, if $a + b = 0$.

10. Factorise: $(3a - 5b)^3 + (4c - 3a)^3 + (5b - 4c)^3$.

Answers

1. (a) -1 **(b)** -52 **(c)** $-25/4$ **2.** $k = \frac{2\sqrt{2} - 1}{4}$ **3.** 1092727

4. $(u-2)(u+2)(3u-4)$.

5. (a) $x^2 + y^2 + 9z^2 - 4xy - 12yz + 6xz$ (b) $\frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$ **6.** -7

7. 322 **8.** $\left(3x - \frac{1}{6}\right)\left(x + \frac{5}{6}\right)$

9. $P = -19$ **10.** $3(3a - 5b)(4c - 3a)(5b - 4c)$

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Previous Years Question Bank

1. If $(x - 1)$ is a factor of $p(x) = x^2 + x + k$, then find the value of k. **[CBSE Schools 2016-17]**

2. Prove that $9x^2 + 30x + 28$ has no zeroes. **[CBSE Schools 2016-17]**

3. Find the value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$, when $a + b + c = 3x$. **[CBSE Schools 2016-17]**

4. Evaluate: $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{c+b} \times \left(\frac{x^c}{x^a}\right)^{a+c}$ **[CBSE Schools 2016-17]**

5. Divide the polynomial $x^4 + x^3 - 2x^2 - x + 1$ by $x + 1$ and verify remainder by using remainder theorem. **[CBSE Schools 2016-17]**

6. What are the possible expressions for the dimensions of a cuboid, whose volume $= 12kx^3 + 34kx^2 + 10kx$? **[CBSE Schools 2016-17]**

7. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$ **[CBSE Schools 2016-17]**

8. Find the remainder when the polynomial $f(x) = x^3 + 4x^2 - 3x + 5$ is divided by $x + 4$. **[CBSE Schools 2016-17]**

9. Without actual calculating the cubes, find the value of $(38)^3 - (12)^3$ **[CBSE Schools 2016-17]**

10. Show that $x + 2$ is a factor of the polynomial $2x^3 + 4x^2 - 3x - 6$. Hence factorise the polynomial. **[CBSE Schools 2016-17]**

11. If $x^4 + \frac{1}{x^4} = 623$, find the value of $x + \frac{1}{x}$, by taking only the positive values of $x + \frac{1}{x}$, $x^2 + \frac{1}{x^2}$, etc. **[CBSE Schools 2016-17]**

12. What should be added to the polynomial $2x^2 - 3x + 3$, so that 1 is a zero of the polynomial? **[CBSE Schools 2016-17]**

13. Expand $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$ **[CBSE Schools 2016-17]**

14. Factorise: $x^3 + 13x^2 + 32x + 10$ **[CBSE Schools 2016-17]**

15. Simplify: $(x + y)^3 - (x - y)^3 - 6y(x + y)(x - y)$ **[CBSE Schools 2016-17]**

16. Verify that $x^3 + y^3 + z^3 - 3xyz = ½ (x + y + z) \left| (x-y)^2 + (y-z)^2 + (z-x)^2 \right|$ **[CBSE Schools 2016-17]**

17. If $a - b = 7$ and $a^2 + b^2 = 85$, find $a^3 - b^3$. **[CBSE Schools 2016-17]**

18. Prove that $\frac{a^{-1}}{a^{-1} + b^{-1}} + \frac{a^{-1}}{a^{-1} - b^{-1}} = \frac{2b^2}{b^2 - a^2}$. **[CBSE Schools 2016-17]**

19. If $p(x) = x^2 + bx - 6$ leaves a remainder 36, when divided by $x - 3$, find the value of 'b' and with this value of b, factorise $p(x)$. **[CBSE Schools 2016-17]**

20. Show by long division that $2x + 3$ is a factor of $p(x) = 4x^4 + 8x^3 + 5x^2 + x - 3$. **[CBSE Schools 2016-17]**

21. Prove that $(x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z + x) = 2(x^3 + y^3 + z^3 - 3xyz)$. **[CBSE Schools 2016-17]**

22. Factorise: $125x^3 + y^3$. **[CBSE Schools 2016-17]**

23. Expand $(x + 2y - 3z)^2$ **[CBSE Schools 2016-17]**

24. If $x^a = y, y^b = z$ and $z^c = x$, then prove that $abc = 1$. **[CBSE Schools 2016-17]**

25. If $(x + a)$ is a factor of each of the polynomials $x^2 + px + q$ and $x^2 + mx + n$, prove that $= \frac{n - q}{m - p}$. **[CBSE Schools 2016-17]**

26. Find what must be subtracted from the polynomial $4y^4+12y^3+6y^2+50y+26$ so that the obtained polynomial is exactly divisible by $y^2 + 4y + 2$. **[CBSE Schools 2016-17]**

27. Find the value of $8a^3 - 27b^3 + 90ab + 125$, if $2a = 3b - 5$. **[CBSE Schools 2016-17]**

28. Examine which of the numbers 1, –1 and –3 are zeroes of the polynomial $2x^4 + 9x^3 + 11x^2 + 4 \times - 6$. **[CBSE Schools 2015–16]**

29. Find the quotient when $x^2 - 7x + 12$ is divided by $x - 3$. **[CBSE Schools 2015–16]**

30. Divide polynomial $p(x) = x^4 - 13x^3 + 29x^2 + 12x - 30$ by $q(x) = x + 1$. Also find what should be subtracted from p(x) so that it is divisible by q(x). **[CBSE Schools 2015–16]**

31. Simplify: $\frac{x^3-4-x+4x^2}{x^2+3x-4}$ **[CBSE Schools 2015–16]**

32. Find the value of 'k'. If $x - 1$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$ **[CBSE Schools 2015–16]**

33. If $x^2 + y^2 = 221$ and $x - y = 1$, then find the value of $x^3 - y^3$. **[CBSE Schools 2015–16]**

34. If $a^2 + b^2 + c^2 = 90$ and $a + b + c = 20$, then find the value of $ab + bc + ca$. **[CBSE Schools 2015–16]**

35. Prove that: $(x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z + x) = 2(x^3 + y^3 + z^3 - 3xyz)$ **[CBSE Schools 2015–16]**

36. Factorise: $27a^3-\frac{1}{64b^3}-\frac{27a^2}{4b}+\frac{9a}{16b^2}$. **[CBSE Schools 2015–16]**

37. Find the product $(5a - 3b)(25a^2 + 15ab + 9b^2)$. **[CBSE Schools 2015–16]**

38. Without actually calculating the cubes, find the value of $(-1)^3 + (-2)^3 + (-3)^3 + (-4)^3 + 2(5)^3$. Also write the identity used. **[CBSE Schools 2015–16]**

39. If $x+\frac{1}{x}=7$, then find the value of $x^3+\frac{1}{x^3}$.

40. Find the value of $(81)^{0.16} \times (81)^{0.09}$ **[CBSE Schools 2014–15]**

41. Factorise : $36x^2 + 60xy + 25y^2$. **[CBSE Schools 2014–15]**

42. If $x^2 + y^2 = 221$ and $x - y = 1$, then find the value of $x^3 - y^3$. **[CBSE Schools 2014–15]**

43. Factorise: $2y^3 + y^2 - 1$. **[CBSE Schools 2014–15]**

44. Prove that : $(x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z + x) = 2(x^3 + y^3 + z^3 - 3xyz)$ **[CBSE Schools 2014–15]**

45. If $a + b + c = 0$, then prove that $a^4 + b^4 + c^4 = 2(b^2c^2 + c^2a^2 + a^2b^2)$ **[CBSE Schools 2014–15]**

46. Express $\frac{x^2-x-6}{x^2-4}$ into lowest terms. **[CBSE Schools 2014–15]**

47. Simplify : $(2x + p - c)^2 - (2x - p + c)^2$. **[CBSE Schools 2014–15]**

48. Factorise : $x^3 + 13x^2 + 32x + 20$. **[CBSE Schools 2014–15]**

49. Divide polynomial $p(x) = x^4 - 13x^3 + 29x^2 + 12x - 30$ by $q(x) = x + 1$. Also what should be subtracted from p(x) so that it is divisible by q(x). **[CBSE Schools 2014–15]**

50. If –2 is a zero of the polynomial $\sqrt{2}\,x+p$ and is also the zero of the polynomial $px^2 + kx + 2 + 2\sqrt{2}$, then find the value of k. **[CBSE Schools 2014–15]**

51. Find the value of m so that $2x - 1$ be a factor of $8x^4 + 4x^3 - 16x^2 + 10x + m$. **[CBSE Schools 2014–15]**

52. Factorise : $27a^3 + 343b^3$.

53. If $x = 2\sqrt{3}-11$, find the value of $x^2+\frac{1}{x^2}$. **[CBSE Schools 2014–15]**

54. Find the value of $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$. **[CBSE Schools 2014–15]**

55. When $x^3 + kx^2 - x + 6$ is divided by $x - 2$ leaves no remainder but when the division $x - 3$, find what remainder it will leave ? **[CBSE Schools 2014–15]**

56. The polynomial $p(x) = ax^3 - 3x^2 + 4$ and $q(x) = 2x^3 - 5x + a$ when divided by $(x - 2)$ leaves remainder as m and n. If $m - 2n = 4$, find a. **[CBSE Schools 2014–15]**

Chapter Test

Maximum Marks: 30 **Maximum Time: 1 hr.**

1. If $x^{40} + x^{51} + 5$ is divisible by $x + 1$, then find the remainder. **[2]**

2. If $f(x) = x^2 - 5x + 1$, then evaluate: $f\ 2 - f\ -1 + f\left(\frac{1}{3}\right)$. **[2]**

3. Using suitable identity, evaluate : 101×102. **[2]**

4. Expand : $\left(4 - \frac{1}{3x}\right)^3$ **[2]**

5. If $x + 1$ is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a. **[2]**

6. Factorise: (i) $36a^3b - 60a^2bc$ (ii) $5x^2 - 20xy$ **[3]**

7. Find the value of $64x^3 - 125y^3$, if **[3]**

$4x - 5y = 16$

and $xy = 12$

8. If $(x - 3)$ and $\left(x - \frac{1}{3}\right)$ are both factors of $ax^2 + 5x + b$, then show that $a = b$. **[3]**

9. Factorise: $a^8 - b^8$. **[3]**

10. Simplify $(a + b)^3 - (a - b)^3 - 6b(a^2 - b^2)$. **[4]**

11. Find the zeroes of the polynomial $f(x) = 2x^3 + 3x^2 - 11x - 6$. **[4]**

Answers

1. 5 **6. (i)** $12a^2b\ (3a - 5c)$ **(ii)** $5x(x - 4y)$ **2.** $-\frac{113}{9}$ **7.** 15616

3. 10302 **9.** $(a-b)(a+b)(a^2+b^2)(a^2+b^2-\sqrt{2}\,ab)(a^2+b^2+\sqrt{2}\,ab)$

4. $65 - \frac{16}{x} + \frac{4}{3x^2} - \frac{1}{27x^3}$ **10.** $8b^3$ **5.** $a = 2$ **11.** 2, –3, –1/2

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