Topic: **Arithmetic Progressions**
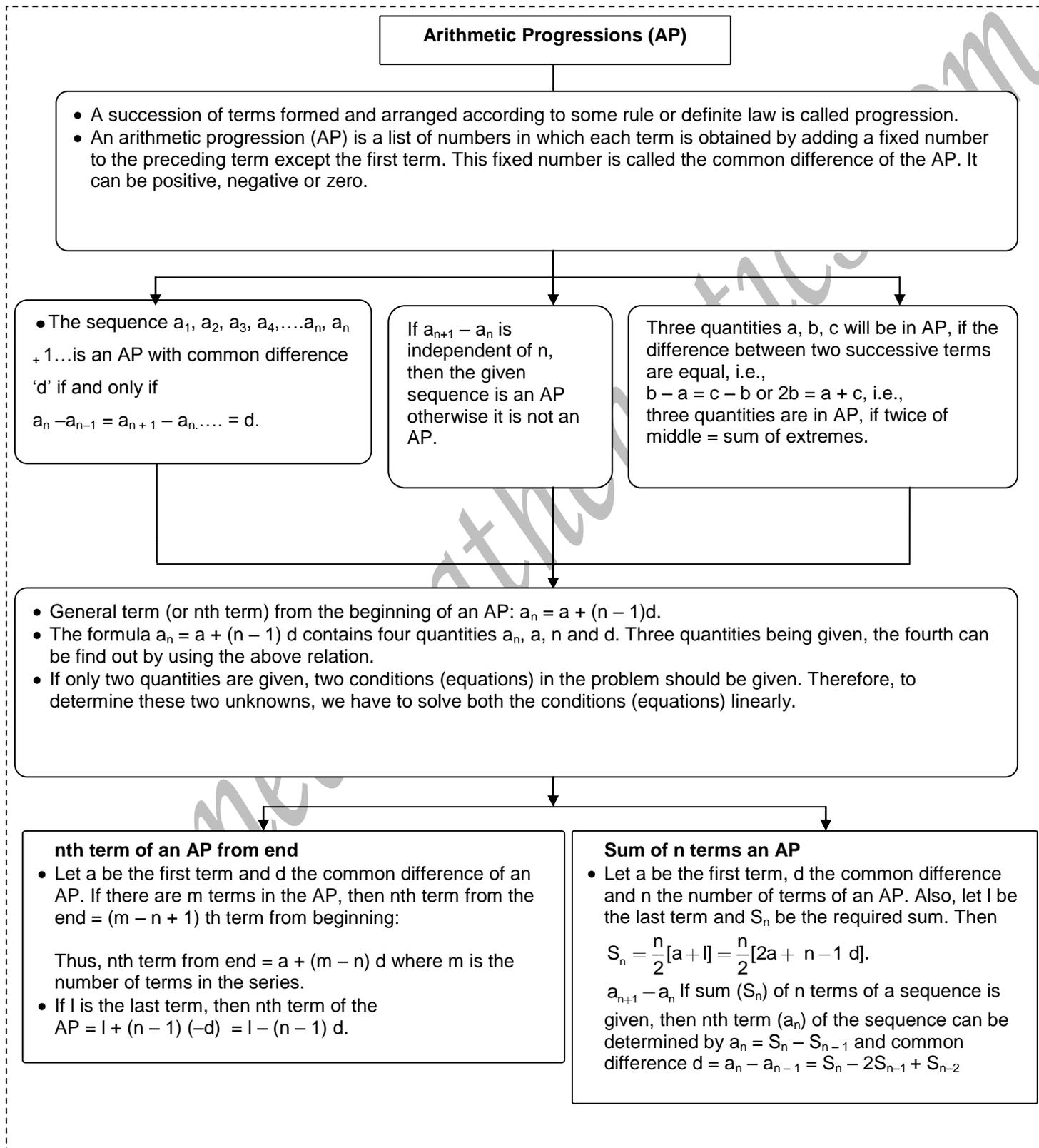
Chapter Flowchart

The Chapter Flowcharts give you the gist of the chapter flow in a single glance.

Arithmetic Progressions (AP)

- A succession of terms formed and arranged according to some rule or definite law is called progression.
- An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. This fixed number is called the common difference of the AP. It can be positive, negative or zero.

- The sequence $a_1, a_2, a_3, a_4, \ldots a_n, a_{n+1} \ldots$ is an AP with common difference 'd' if and only if

  $a_n - a_{n-1} = a_{n+1} - a_n \ldots = d$.

If $a_{n+1} - a_n$ is independent of n, then the given sequence is an AP otherwise it is not an AP.

Three quantities a, b, c will be in AP, if the difference between two successive terms are equal, i.e.,
b – a = c – b or 2b = a + c, i.e., three quantities are in AP, if twice of middle = sum of extremes.

- General term (or nth term) from the beginning of an AP: $a_n = a + (n - 1)d$.
- The formula $a_n = a + (n - 1)d$ contains four quantities a_n, a, n and d. Three quantities being given, the fourth can be find out by using the above relation.
- If only two quantities are given, two conditions (equations) in the problem should be given. Therefore, to determine these two unknowns, we have to solve both the conditions (equations) linearly.

nth term of an AP from end
- Let a be the first term and d the common difference of an AP. If there are m terms in the AP, then nth term from the end = (m – n + 1) th term from beginning:

  Thus, nth term from end = a + (m – n) d where m is the number of terms in the series.
- If l is the last term, then nth term of the AP = l + (n – 1) (–d) = l – (n – 1) d.

Sum of n terms an AP
- Let a be the first term, d the common difference and n the number of terms of an AP. Also, let l be the last term and S_n be the required sum. Then

  $S_n = \dfrac{n}{2}[a + l] = \dfrac{n}{2}[2a + \overline{n - 1}\, d]$.

  $a_{n+1} - a_n$ If sum (S_n) of n terms of a sequence is given, then nth term (a_n) of the sequence can be determined by $a_n = S_n - S_{n-1}$ and common difference $d = a_n - a_{n-1} = S_n - 2S_{n-1} + S_{n-2}$

Revision Question Bank

Subjective Types Questions

1. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

2. How many terms of the series 54, 51, 48............be taken so that, their sum is 513? Explain the double answer.

3. If S_n denotes the sum of first n terms of an AP, prove that : $S_{12} = 3 \ S_8 - S_4$

4. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

5. The sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Determine the AP and the 12th term.

6. The ratio of the sums of m and n terms of an AP is $m^2 : n^2$. Show that the ratio of the mth and nth terms is $(2m - 1) : (2n - 1)$.

7. Divya deposited Rs 1000 at compound interest at the rate of 10% per annum. The amounts at the end of first year, second year, third year, ..., form an AP. Justify your answer.

8. For the AP : –3, –7, –11, ..., can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20} ? Give reasons for your answer.

9. If the sum of m terms of an AP is the same as the sum of its n terms, show that the sum of its (m + n) terms is zero.

10. Kanika was given her pocket money on Jan 1st, 2008. She puts Re 1 on Day 1, Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end of the month she still had Rs 100 with her. How much was her pocket money for the month?

Answers

1. $a_{22} = 64$ **2.** The sum of 18 terms as well as that of 19 terms is 513

4. 3, 7, 11, 15,.....   **5.** $A_{12} = 65$ **7.** It doesn't from an A.P.

8. yes, $a_{30} - a_{20} = (30 - 20)d = 10d = -40$ **9.** $S_{m+n} = 0$

10. Her pocket many was Rs. 800

Previous Years Question Bank

1. The first and last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum? **[CBSE Schools, 2016-17]**

2. An arithmetic sequence has its fifth term equal to 22 and its 15th term equal to 62. Find its hundredth term and sum of first 50 terms. **[CBSE Schools, 2016-17]**

3. The sum of first three terms of an AP is 45. If the product of the first and the third terms exceeds the second term by 21, find the AP. **[CBSE Schools, 2016-17]**

4. Find the number of terms in an A.P. 6, 10, 14, ..., 82. **[CBSE Schools, 2016-17]**

5. If four times the 4th term of an A.P. is equal to eight times its 8th term, show that its 12th term is zero.

6. Find the sum of first 24 terms of an A.P. whose nth term is given by a_n = 3 + 2n. **[CBSE Schools, 2016-17]**

7. Find the sum of all natural numbers between 1000 and 10000. **[CBSE Schools, 2016-17]**

8. Find the value of x if 12th term of the A.P.: x – 7, x –2, x + 3, ...is 81. Find S_{12} also. **[CBSE Schools, 2016-17]**

9. The ratio of the 5th and 3rd terms of an AP is 2.5. Find the ratio of the 15th and 7th terms.

[CBSE Schools, 2016-17]

10. Find if 100 is a term of the A.P., 25, 28, 31, ... or not. **[CBSE Schools, 2015-17]**

11. Write first four terms of the AP, when the first term is 1.25 and common difference is -0.25.

[CBSE Schools, 2016-17]

12. How many terms of the AP : 9, 17, 25, ... Must be taken to get a sum of 450? **[CBSE Schools, 2016-17]**

13. Find the sum of the integers between 100 and 200 that are **[CBSE Schools, 2016-17]**
(i) divisible by 6.   (ii) not divisible by 6.

14. Find the 20th term from the last term of the A.P. : 3, 8, 13, ..., 253. **[CBSE Schools, 2016-17]**

15. Solve the equation: 1 + 4 + 7 + 10 + ... + x = 287. **[CBSE Schools, 2016-17]**

16. If the 19th term of an AP is 47, then find the sum of the first 37 terms. **[CBSE Schools, 2016-17]**

17. Find the sum of all natural numbers amongst first one thousand numbers which are neither divisible by 2 nor by 5. **[CBSE Schools, 2016-17]**

18. In an AP, 6th term is half the 4th term, and the 3rd term is 15. How many terms are needed to give a sum that is equal to 66? **[CBSE Schools, 2016-17]**

19. What is the common difference of an A.P. in which $a_{21} - a_7$ = 84? **[CBSE Schools, 2016-17]**

20. Which term of the progression 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$,.... is the first negative term? **[CBSE Schools, 2016-17]**

21. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P. **[CBSE Schools, 2016-17]**

22. If the ratio of the sum of the first n terms of two A.Ps is (7n + 1) : (4n + 27), then find the ratio of their 9th terms. **[CBSE Schools, 2016-17]**

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23. If five terms the fifth term of an A.P. is equal to eight times its eighth term, show that its 13th term is zero.

[CBSE Schools, 2016-17]

24. The sum of the first 8 terms of an AP is 100 and the sum of its first 19 terms is 551. Find the first term and the common difference of the AP. **[CBSE Schools, 2016-17]**

25. If pth term of an A.P is 2p–1, then find the common difference. **[CBSE Schools, 2015-16]**

26. For what value of n, are the nth terms of two A.Ps : 63, 65, 67, ...and 3, 10, 17, ... are equal?

[CBSE Schools, 2015-16]

27. Find the sum of first 25 terms of an A.P, in which the third term is 7 and seventh term is two more than thrice of its third term. **[CBSE Schools, 2015-16]**

28. If the common difference of an AP is 3, then what is $a_{15} - a_9$? **[CBSE Schools, 2015-16]**

29. If 8th term of an AP is 37 and 12th term is 57, then find the AP. **[CBSE Schools, 2015-16]**

30. For what value of k will k+9, 2k-1 and 2k+7 are the consecutive terms of an A.P.?

[CBSE Schools, 2015-16]

31. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

[CBSE Schools, 2015-16]

32. The sums of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P. is 1 and their common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_2 = 2S_2$.

[CBSE Schools, 2015-16]

33. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X. **[CBSE Schools, 2015-16]**

34. The 13th term of an AP is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms. **[CBSE Schools, 2014-15]**

35. An arithmetic progression 5, 12, 19,...has 50 terms. Find its last term. Hence find the sum of its last 15 terms. **[CBSE Schools, 2014-15]**

36. The sum of *n* terms of an AP is $(2n + 3n^2)$. Determine the AP and find its rth term.

[CBSE Schools, 2014-15]

37. In an AP, 6th term is half the 4th term, and the 3rd term is 15, How many terms are needed to give a sum that is equal to 66? **[CBSE Schools, 2014-15]**

38. Mr. John runs a scooter manufacturing factory. His factory produces 1100 scooters in the third year and the production in the eleventh year is 2700. If The production increases by a steady rate every year, find the production of scooters in the 25^{lh} year and the total number of scooters produced in all these 25 years. **[CBSE Schools, 2014-15]**

39. Which term of the A.P. 3,15,27,39........will be 120 more than its 21st term? **[CBSE Schools, 2014-15]**

40. The first term of the A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms' and common difference? **[CBSE Schools, 2014-15]**

41. A spiral is made up of successive semi circles, with centers alternatively at A and B, starting. with the center A, of radii 0.5 cm, 1.0cm, 1.5cm, 2.0cm,...........in Fig. **[CBSE Schools, 2014-15]**
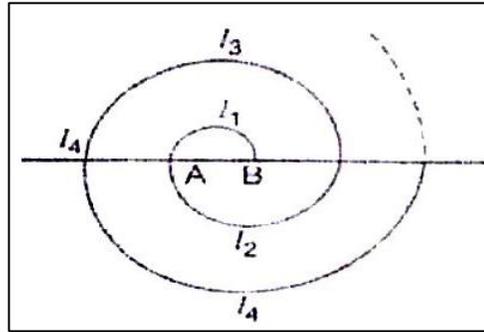
What is the total length of such a spiral made up of 13 consecutive semi circles (Take ,π = 22/7)

42. The sum of the 5th and the 9th terms of an AP is 30. If its 25th term is three times its 8th term, find the AP. **[CBSE Schools, 2013-14]**

43. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. It there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this equation? **[CBSE Schools, 2013-14]**

44. The sum of three numbers in A.P. is 21 and their product is 231. Find the numbers. **[CBSE Schools, 2011-12]**

45. The ratio of the 7th term to the 3rd term of an A.P. is 12:5. Find the ratio of 13th term to the 4th term.

46. If the sum of first 7 terms of an A.P. is 49 and that of first 17 terms is 289, then find the sum of the first n terms of the A.P. **[CBSE Schools, 2011-12]**

47. A sum of Rs. 2200 is to be used to give eight cash prizes to students of a school for their overall academic performance. If each prize is Rs. 50 less than its preceding price, find the value of each of the prizes. **[CBSE Schools, 2011-12]**

Chapter Test

Maximum Marks: 30   **Maximum Time: 1 hour**

1. What is the common difference of the AP $\dfrac{1}{b}, \dfrac{3-b}{3b}, \dfrac{3-2b}{3b}, ...$?   **[1]**

2. Find the sum of first 'n' terms of the series a, 3a, 5a,....   **[1]**

3. Which term of the AP:3, 15, 27, 39,.... will be 120 more than its 21st term?   **[2]**

4. Which term of the AP: 150, 147, 144,.... is its first negative term?   **[2]**

5. Find the sum of all the two-digit natural numbers which are divisible by 4.   **[3]**

6. If a, b, c are in A.P. show that $\dfrac{1}{\sqrt{b}+\sqrt{c}}, \dfrac{1}{\sqrt{c}+\sqrt{a}}, \dfrac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.   **[3]**

7. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 135$, then find the AP, where S_n denotes the sum of its first n terms.

[4]

8. A spiral is made up of successive semicircles, with centre alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ...in Fig. What is the total length of such a spiral made up of thirteen consecutive semi-circles? (Take $\pi = \dfrac{22}{7}$)   **[4]**
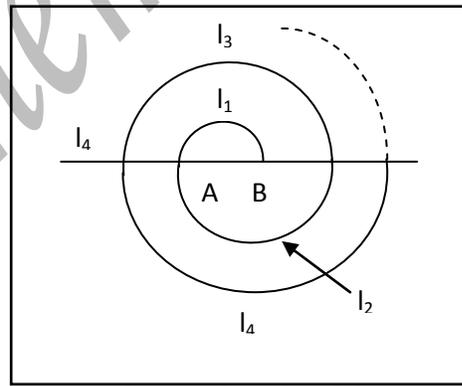
9. A sum of Rs 3150 is to be used to give six cash prizes to students of a school for overall academic performance, punctuality, regularity, cleanliness, confidence, and creativity. If each prize is Rs 50 less than its preceding prize, find the value of each of the prizes.

(a) Which value according to you should be awarded with maximum amount ? Justify your answer.

(b) Can you add more values to the above ones which should be awarded ?   **[4]**

10. The 13th term of an AP is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms.   **[4]**

Answers
1. –1/3   **2.** an^2 **3.** 31st term   **4.** 52nd term   **5.** 1188
7. 1, 6, 11, 16, -----   **8.** 143 cm   **9.** 157
10. 650, 600, 550, 450, 500 (a) Any value with justification is correct
(b) Many more can be added like honesty, good habits, friendship, respecting elders etc.