Trigonometry

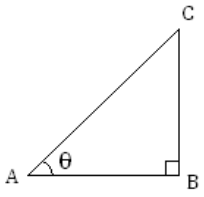
Chapter Flowchart

The Chapter Flowcharts give you the gist of the chapter flow in a single glance.

Trigonometry is the branch of mathematics which deals with the measurements of sides and angles of the triangles

Trigonometry Ratios

Let ABC be a right triangle, right-angled at B, Let $\angle CAB = \theta$,
Then,



$$\sin\theta = \frac{BC}{AC} \quad \cos\theta = \frac{AB}{AC} \quad \tan\theta = \frac{BC}{AB}$$

$$\cot\theta = \frac{AB}{BC} \quad \sec\theta = \frac{AC}{AB} \quad \operatorname{cosec}\theta = \frac{AC}{BC}$$

Relation between Trigonometry Ratios

Reciprocal Relations

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} \Rightarrow \operatorname{cosec}\theta = \frac{1}{\sin\theta} \Rightarrow \sin\theta.\operatorname{cosec}\theta = 1$$

$$\cos\theta = \frac{1}{\sec\theta} \Rightarrow \sec\theta = \frac{1}{\cos\theta} \Rightarrow \sec\theta.\cos\theta = 1$$

$$\tan\theta = \frac{1}{\cot\theta} \Rightarrow \cot\theta = \frac{1}{\tan\theta} \Rightarrow \tan\theta.\cot\theta = 1$$

Quotient Relations

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trigonometric Identities

some common trigonometric identities are

(i) $\sin^2\theta + \cos^2\theta = 1$ for $0^0 \le \theta \le 90^0$

(ii) $\sec^2\theta = 1 + \tan^2\theta$ for $0^0 \le \theta \le 90^0$

(iii) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ for $0^0 \le \theta \le 90^0$

Trigonometric ratios of complementary angles

(i) $\sin(90^0 - \theta) = \cos\theta$ (ii) $\cos(90^0 - \theta) = \sin\theta$

(iii) $\tan(90^0 - \theta) = \cot\theta$ (iv) $\cot(90^0 - \theta) = \tan\theta$

(v) $\sec(90^0 - \theta) = \operatorname{cosec}\theta$ (vi) $\operatorname{cosec}(90^0 - \theta) = \sec\theta$

Values of trigonometric ratios of standard angles

θ	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec}\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Revision Question Bank

1. Evaluate $\frac{\sec^2\ 90^0-\theta\ -\cot^2\theta}{2\ \sin^2 25^0+\sin^2 65^0}+\frac{2\cos^2 60^0\tan^2 28\ \tan^2 62^0}{3\ \sec^2 43^0-\cot^2 47^0}+\frac{\cot 40^0}{\tan 50^0}$

2. Show that: $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$.

3. Given that sin (A + B) = sin A cos B + cos A sin B, find the value of $\sin 75^0$.

4. Without using trigonometric tables, evaluate:

$\frac{\cos^2 20^0+\cos^2 70^0}{\sec^2 50^0-\cot^2 40^0}$ × $\sec^2 60^0$ – 2 $\cot 58^0 \cot 32^0$ – 4 $\tan 13^0 \tan 37^0 \tan 45^0 \tan 53^0 \tan 77^0$

5. Prove the identity: $\frac{1+\cos A}{1-\cos A}-\frac{1-\cos A}{1+\cos A}=4\cot A\,\text{cosec}$.

6. Show that: (cosec A –1) (cosec A +1) (sec A –1) (sec A +1) =1.

7. Show that: $\frac{2-\text{cosec}^2 A}{\text{cosec}^2 A+2\cot A}=\frac{\sin A-\cos A}{\sin A+\cos A}$.

8. If 3 cot A = 4 , then check whether

$\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$ or not.

9. If sin (A – B) = $\frac{1}{2}$ and cos (A + B) = $\frac{1}{2}$, $0^0 < A + B \le 90^0$, A < B, find A and B.

10. If each of α, β and γ is a positive acute angles such that $\sin\ \alpha+\beta-\gamma\ =\frac{1}{2}$, $\cos\ \beta+\gamma-\alpha\ =\frac{1}{2}$ and $\tan\ \gamma+\alpha-\beta\ =1$, find the value of α, β and γ.

Answers

1. 5/3 **3.** $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **4.** 2 **8.** Yes

9. A = 45^0, B = 15^0. **10.** $\alpha = 37\frac{1}{2}^0$, $\beta = 45^0$, $\gamma = 52\frac{1}{2}^0$

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Previous Years Question Bank

1. Find the value of $4\,\text{cosec}^2 60^0 - 16\tan^2 30^0$. **[CBSE Schools 2016-17]**

2. If $2\sin(90^0 - \phi)\cos\phi = 1$, then find the value of ϕ. **[CBSE Schools 2016-17]**

3. If $\tan(3x - 15^0) = 1$, then find the value of x. **[CBSE Schools 2016-17]**

4. In ΔABC, $\angle A = 90^0$. If $\tan B = \sqrt{3}$, find the value of $\sin B\cos C + \cos B\sin C$. **[CBSE Schools 2016-17]**

5. Prove the identity: $\dfrac{1}{\sec x + \tan x} - \dfrac{1}{\cos x} = \dfrac{1}{\cos x} - \dfrac{1}{\sec x - \tan x}$ **[CBSE Schools 2016-17]**

6. If $n\sin\theta = m\cos\theta$, then prove that

$$\frac{m\sin\theta - n\cos\theta}{m\sin\theta + n\cos\theta} + \frac{m\sin\theta + n\cos\theta}{m\sin\theta - n\cos\theta} = \frac{2\,m^4 + n^4}{m^4 - n^4}.$$ **[CBSE Schools 2016-17]**

7. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$. **[CBSE Schools 2016-17]**

8. Prove that: $\dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$ **[CBSE Schools 2016-17]**

9. Find the value of $\sin 38° - \cos 52°$. **[CBSE Schools 2016-17]**

10. If $x = 3\sin\theta$ and $y = 4\cos\theta$, find the value of $\sqrt{16x^2 + 9y^2}$. **[CBSE Schools 2016-17]**

11. Prove the following identity: $\left(1 + \dfrac{1}{\tan^2 A}\right).\left(1 + \dfrac{1}{\cot^2 A}\right) = \dfrac{1}{\cos^2 A - \cos^4 A}$. **[CBSE Schools 2016-17]**

12. If $\theta = 60°$, show that:

(i) $\sin\theta = \dfrac{\tan\theta}{\sqrt{1 + \tan^2\theta}}$ (ii) $\tan\theta = \dfrac{\sqrt{1 - \cos^2\theta}}{\cos\theta}$ **[CBSE Schools 2016-17]**

13. Prove that: $\left(\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A}\right).\left(\dfrac{\cos A}{1 - \sin A} - \dfrac{1 - \sin A}{\cos A}\right) = 4\tan A.\sec A$ **[CBSE Schools 2016-17]**

14. In ΔPQR, $\angle Q = 90^0$ and $\sin R = \dfrac{3}{5}$, write the value of $\cos P$. **[CBSE Schools 2016-17]**

15. Prove that: $\dfrac{1}{1 + \sin A} + \dfrac{1}{1 - \sin A} = 2\sec^2 A$ **[CBSE Schools 2016-17]**

16. Evaluate the following: **[CBSE Schools 2016-17]**

$$\cot\theta.\tan\left(90^0 - \theta\right) - \sec\left(90^0 - \theta\right).\text{cosec}\,\theta + \sin^2 65^0 + \sin^2 25^0 + \sqrt{3}\tan 50.\tan 45^0.\tan 85^0.$$

17. Prove the identity: $\sec^2\theta\left(\sec^2\theta - 2\right) + 1 = \tan^4\theta$. **[CBSE Schools 2016-17]**

18. Prove that: $\dfrac{\sin^3\theta}{\cot\theta} + \dfrac{\cos^3\theta}{\tan\theta} + 2\sin\theta.\cos\theta = \sec\theta.\text{cosec}\,\theta = \tan\theta + \cot\theta$ **[CBSE Schools 2016-17]**

19. Prove that: $(1 + \cot A + \tan A).(\sin A - \cos A) = \dfrac{\sec^3 A - \operatorname{cosec}^3 A}{\sec^2 A - \operatorname{cosec}^2 A}$ **[CBSE Schools 2016-17]**

20. Find the value of $3\sin\alpha - 4\cos^3\alpha$, if $\sin\alpha = \dfrac{1}{2}$ **[CBSE Schools 2016-17]**

21. Evaluate: $\operatorname{cosec} 39^0 . \cos 51^0 + \tan 21^0 . \cot 69^0 - \sec^2 21^0$ **[CBSE Schools 2016-17]**

22. Prove that; $\dfrac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \left(\dfrac{1+\sin\theta}{\cos\theta}\right)^2$ **[CBSE Schools 2016-17]**

23. If $A + B = 90^0$, prove that: $\sqrt{\dfrac{\tan A + \tan B + \tan A \cot B}{\sin A \sec B} - \dfrac{\sin^2 B}{\cos^2 A}} = \tan A$ **[CBSE Schools 2016-17]**

24. If $l\sin\theta + m\cos\theta + n = 0$ and $l'\sin\theta + m'\cos\theta + n' = 0$,

then prove that $(mn' - m'n)^2 + (n\,l' - ln')^2 = (l\,m' - l'm)^2$. **[CBSE Schools 2016-17]**

25. Prove that: $\tan\theta + \sec\theta - 1 \;.\; \tan\theta + 1 + \sec\theta = \dfrac{2\sin\theta}{1-\sin\theta}$ **[CBSE Schools 2016-17]**

26. If $\sec\theta . \sin\theta = 0$, then find value of θ. **[CBSE Schools 2016-17]**

27. Prove that: $\tan^2 A + \cot^2 A + 2 = \sec^2 A . \operatorname{cosec}^2 A$ **[CBSE Schools 2016-17]**

28. Prove that: $\left(\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A}\right).\left(\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}\right) = 4 \sec A . \operatorname{cosec} A$ **[CBSE Schools 2016-17]**

29. If $\sin 2A = \cos(A - 30°)$, where $2A$ is an acute angle, find the value of A. **[CBSE Schools 2016-17]**

30. Find the value of: $\sin 30° \cos 60° + \cos 30° \sin 60°$ **[CBSE Schools 2016-17]**

Show it equal to $\sin 90^0$ or $\cos 90^0$?

31. Prove the identity $\sin^2\theta + \cos^2\theta = 1$ and use it to prove $\sin^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

[CBSE Schools 2016-17]

32. Simplify: $\dfrac{1+\tan^2 A}{1+\cot^2 A}$ **[CBSE Schools 2016-17]**

33. If $\tan 2A = \cot(A + 60°)$, find the value of A where $2A$ is an acute angle. **[CBSE Schools 2016-17]**

34. If $4\sec\theta = 5$, then evaluate : $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$ **[CBSE Schools 2016-17]**

35. Prove that: $1+\cot^2\theta \;.\; 1+\cos\theta \;.\; 1-\cos\theta = 1+\tan^2\theta \;.\; 1+\sin\theta \;\; 1-\sin\theta = 1$ **[CBSE Schools 2016-17]**

36. Express sinA, cosA, cosec A and sec A in terms of cot A. **[CBSE Schools 2016-17]**

37. $\dfrac{\sec\alpha}{\sec\beta} = m$ and $\dfrac{\sec\alpha}{\operatorname{cosec}\beta} = n$, show that $m^2 + n^2 = n^2 \operatorname{cosec}^2\beta$. **[CBSE Schools 2016-17]**

38. Prove that: $\dfrac{\cos A}{1+\sin A} + \dfrac{\cos A}{1-\sin A} = \sqrt{\dfrac{1-\sin A}{1+\sin A}} + \sqrt{\dfrac{1+\sin A}{1-\sin A}} = 2\sec A$ **[CBSE Schools 2016-17]**

39. If $\sec(7^0 - 2\alpha) = \operatorname{cosec}(5\alpha - 7^0)$, then find the value of α **[CBSE Schools 2016-17]**

40. Simplify: $\frac{1}{\sec^2\theta} + \frac{1}{\operatorname{cosec}^2\theta}$. **[CBSE Schools 2016-17]**

41. If $\sin(2x + 3y) = 1$; $\cos(2x - 3y) = \frac{\sqrt{3}}{2}$, find the values of x and y. **[CBSE Schools 2016-17]**

42. Prove that: $\frac{1+\sin\theta^{\ 2} + \ 1-\sin\theta^{\ 2}}{2\cos^2\theta} = \frac{1+\sin^2\theta}{1-\sin^2\theta}$ **[CBSE Schools 2016-17]**

43. Prove that: $\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1+\cos\theta}{\sin\theta}$ **[CBSE Schools 2016-17]**

44. Simplify : $\frac{\tan 28^0}{\cot 62^0} \div \frac{1}{\sqrt{3}}$ [$\tan 20^0$, $\tan 60^0$. $\tan 70^0$] **[CBSE Schools 2015-16]**

45. If $\tan(20^0 - 3\alpha) = \cot(5\alpha - 20^0)$, then find the value of α and hence evaluate:

$\sin\alpha \cdot \sec\alpha \cdot \tan\alpha - \operatorname{cosec}\alpha \cdot \cos\alpha \cdot \cot\alpha$. **[CBSE Schools 2015-16]**

46. Prove that: $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = (7 + \tan^2\theta + \cot^2\theta)$. **[CBSE Schools 2015-16]**

47. Prove that: $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$. **[CBSE Schools 2015-16]**

48. Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$ **[CBSE Schools 2015-16]**

49. If p = sec A + tan A, then prove that $\sin A = \frac{p^2 - 1}{p^2 + 1}$ **[CBSE Schools 2015-16]**

50. If cos A – sin A = m and cos A + sin A = n, show that $\frac{m^2 - n^2}{m^2 + n^2} = -2\sin A.\cos A = -\frac{2}{\tan A + \cot A}$ **[CBSE Schools 2015-16]**

51. Prove that following identity: $\left[\frac{1 - \tan A}{1 - \cot A}\right]^2 = \tan^2 A; \angle A$ is acute. **[CBSE Schools 2014-15]**

52. Prove that: $(1 + \cot\theta - \operatorname{cosec}\theta).(1 + \tan\theta + \sec\theta) = 2$ **[CBSE Schools 2014-15]**

53. If $\cot\theta = 3x - \frac{1}{12x}$, then show that $\cot\theta + \operatorname{cosec}\theta = 6x$ or $-\frac{1}{6x}$. **[CBSE Schools 2014-15]**

54. Prove that: $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A}$ **[CBSE Schools 2014-15]**

55. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sqrt{mn}$. **[CBSE Schools 2014-15]**

56. Prove that: $\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta = 2\sin^2\theta - 1 = 1 - 2\cos^2\theta$. **[CBSE Schools 2014-15]**

57. Prove that: $\frac{\tan\theta + \sin\theta}{\tan\theta - \tan\theta} = \frac{\sec\theta + 1}{\sec\theta - 1} = \frac{1 + \cos\theta}{1 - \cos\theta}$. **[CBSE Schools 2014-15]**

58. Prove that $\frac{\sin A}{\cot A + \operatorname{cosec} A} = 2 + \frac{\sin A}{\cot A - \operatorname{cosec} A}$.

59. If $x = a \sin\theta$ and $y = b \tan\theta$. Prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2}$. **[CBSE Schools 2014-15]**

60. Evaluate: $\frac{\cos 70^0}{\sin 20^0} + \frac{\cos 55^0 \operatorname{cosec} 35^0}{\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0}$. **[CBSE Schools 2014-15]**

61. Prove the following identity: $\left[\frac{12\tan A}{12\cot A}\right]^2 = \tan^2 A$; $\angle A$ acute. **[CBSE Schools 2014-15]**

62. For any acute angle θ, prove that: **[CBSE Schools 2014-15]**

(i) $\sin^2\theta \cos^2\theta = 1$ (ii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$.

Chapter Test

Maximum Marks: 30 **Maximum Time: 1 hr.**

1. Without using trigonometrical tables, evaluate

$$\frac{\cos 58^0}{\sin 32^0}+\frac{\sin 22^0}{\cos 68^0}-\frac{\cos 38^0}{\tan 18^0 \tan 35^0 \tan 60^0 \tan 72^0 \tan 55^0}$$ **[3]**

2. If $7\sin^2\theta+3\cos^2\theta=4$ and θ is an acute angle, then prove that: $\sec\theta+\operatorname{cosec}\theta=2+\frac{2}{\sqrt{3}}$. **[3]**

3. Find the acute angles of A and B, if sin (A + 2B) = $\frac{\sqrt{3}}{2}$ and cos (A + 4B) = 0^0, where A > B. **[3]**

4. If $(\cot\theta+\tan\theta)=m$ and $(\sec\theta-\cos\theta)=n$, then prove that: $mn^{2/3}-mn^{2/3}=1$. **[3]**

5. Without using trigonometric tables evaluate the following

$$\frac{\operatorname{cosec}^2 90^0-\theta-\tan^2\theta}{4\cos^2 48^0+\cos^2 42^0}-\frac{2\tan^2 30^0\sec^2 52^0\sin^2 38^0}{\operatorname{cosec}^2 70^0-\tan^2 20^0}$$ **[3]**

6. If $\cos\alpha=\frac{1}{2}$ and $\tan\beta=\frac{1}{\sqrt{3}}$ then find, Sin $\alpha+\beta$ where α and β are both acute angles. **[3]**

7. Prove that : $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\cot\theta+\tan\theta$. **[4]**

8. Prove that : $\frac{1}{\operatorname{cosec}\theta-\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta+\cot\theta}$. **[4]**

9. Prove that: $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$ **[4]**

Answers

1. $\frac{2\sqrt{3}-1}{\sqrt{3}}$ **3.** A = 30^0, B = 15^0 **4.** $\sec\theta-\cos\theta$

5. $-5/12$ **6.** 1

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