

Pair of linear equations in two variables

Chapter Flowchart

The Chapter Flowcharts give you the gist of the chapter flow in a single glance.

Pair of linear equations in two variables

Algebraic expression: A combinations of constants and variable, connected by four fundamental arithmetical operation of +, -, \times and \div is called an algebraic expression.
For example, $3x^3 + 4xy - 5y^2$ is an algebraic expression.

Equation : An algebraic expression with equal to sign(=) is called the equation. Without an equal to sign, it is an expression only.
For example, $3x + 9 = 0$ and but $3x + 9$ is an expression only.

Linear equation: If the greatest exponent of the variable (s) in a equation is one, then equation is said to be a linear equation.

1. If the number of variables used in linear equation in one, then equation is said to be linear equation in one variable.

For example, $3x + 4 = 0$, $3y + 15 = 2t + 15 = 0$; and so on

2. If the number of variable used in linear equation is two, then equation is said to be linear equation in two variables.

For example, $3x + 2y = 12$; $4c + 6z = 24$, $3y + 4t = 15$, etc.

Thus, equation of the form $ax + by + c = 0$, where a, b, and c are real numbers ($a, b \neq 0$) is called a linear equation in two variables.

Solution: Solution(s) is /are the value/values for the variable (s) used in equation which make(s) the two side of the equation equal.

1. Two linear equations of the form $ax + by + c = 0$, taken together form a system of linear equations, and pair of values of x and y satisfying each one of the given equation is called a solution of the system.

2. To get the solution of simultaneous linear equations, two methods are used:

(i) Graphical method (ii) Algebraic method

Graphical method

Suppose
 $a_1x + b_1x + c_1 = 0$ (i)
 $a_2x + b_2x + c_2 = 0$ (ii)
 be a system of simultaneous linear equations in variables
 x and y .

Graphical method

(a) In order to represent given pair of linear equations graphically, we will have to find two points on the line represented by equation i.e. we will have to find two solutions of each equation.

(b) Every solution $x = a, y = b$ (where a and b are real numbers), of the given equation determines a point (a, b) which lies on the graph of line.

(c) Every point (c, d) lying on the line determines a solution $x = c, y = d$ of the given equation. Thus, line is known as the graph of the given equation.

(d) The graph of a pair of linear equations in two variables is represented by two lines.

(i) If the lines intersect at a point, then that point gives the unique solution of the equations.

In case, the pair of equations is consistent.

(ii) If the lines coincide, then there are infinitely many solutions – each point on the line being a solution. In this case, the pair of equations is dependent (consistent)

(iii) If the lines are parallel, then the pair of equations has no solution. In case, the pair of equations is inconsistent.

(i) Substitution Method

- Choose either of the two equations, say (i), and find the value of one variable, say y , in terms of x .
- Substitute the value of y , in the other equation i.e. (ii)
- Solve the equation to get the value of x .
- Substitute the value of x in the expression for y in terms of x to get the value of y .

(ii) Method of Elimination

- Multiply the equations so as to make the coefficients of the variable to be eliminated equal.
- Add or subtract the equations according as the terms having the same coefficients are of the same sign.
- Solve equation in one variable.
- Substitute the value found in any one of the given equations and find the value of the other variable.

(iii) Cross multiplication

- To obtain the solution, write x , $-y$ and one separated by equality signs as shown:

$$\frac{x}{b_1} = \frac{-y}{a_1} = \frac{1}{a_1 b_2 - a_2 b_1}$$

- The arrows between two numbers indicate that the numbers are to be multiplied.
- Applying this, we get

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{-y}{a_1 c_2 - a_2 c_1} = \frac{1}{a_1 b_2 - a_2 b_1}$$
- Obtain the value of x by equating first and third expression. The value of y is obtained by equating second and third expression.

Equations reducible to a pair of linear equations in two variables

Sometimes there are pairs of equations which are not linear but they can be reduced to linear form by making some suitable substitutions. After converting them in the linear form, we can easily find their solution by any of the three methods.

Conditions for solvability (or consistency)

If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise.

- (i) $\frac{a_1}{b_1} \neq \frac{a_2}{b_2}$, In this case, the pair of linear equations has a unique solution (consistent pair of equations)
- (ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, In this case, the pair of linear equations has no solution (inconsistent pair of equations)
- (iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, In this case, the pair of linear equations has infinitely many solutions [(dependent (consistent) pair of equations)].

Revision Question Bank

1. Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.
2. Solve the following system of linear equations
 $8v - 3u = 5uv$ and $6v - 5u = -2uv$.
3. Solve for x and y by using the method of elimination
 $0.4x + 0.3y = 1.7$ and $0.7x - 0.2y = 0.8$.
4. Solve the following system of linear equations by the method of cross-multiplication
 $2x - y - 3 = 0$ and $4x + y - 3 = 0$.
5. Solve the following system of equations in x and y
 $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ and $(a + b)(x + y) = a^2 + b^2$.
6. A Lending library has a fixed charge for the first three days and an additional charge for each day, thereafter and Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.
7. For the following system of equations, determine the value of k for which the given system of equations has a unique solution. $2x + 3y - 5 = 0$ and $kx - 6y - 8 = 0$
8. Five years ago, Ramesh was thrice of Shyam's age. Ten years later, Ramesh will be twice of Shyam's age. How old are Ramesh and Shyam?
9. Father's age is 3 times the sum of ages of his two children. After 5 yr, his age will be twice the sum of ages of the two children. Find the age of father.
10. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Find the area of this triangle.

Answers

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|---|--|
| 1. (2, 3), (-1, 0), (4, 0). | 2. $u = 0$ and $v = 0$, $u = \frac{22}{23}$ and $v = \frac{11}{23}$. |
| 3. $x = 2$, $y = 3$. | 4. $x = 1$, $y = -1$. |
| 5. $x = a + b$, $y = -\frac{2ab}{a + b}$. | 6. Rs. 15, Rs. 3. |
| 7. $k = -4$ | 8. Ramesh's age = 50 yr, Shyam's age = 20 yr. |
| 9. 45 yr. | 10. (0, 4), (2, 0), (0, -4), 8sq. units. |

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Previous years Question Bank

1. Solve the following pair of linear equations graphically : **[CBSE Schools 2016-2017]**
 $2x + y = 4$
 $2x - y = 4.$
Also, find the coordinates of the vertices of the triangle formed by the lines with y-axis and also find the area of triangle.
2. Determine graphically whether the following pair of linear equations **[CBSE Schools 2016-2017]**
 $4x + 6y = 96$
 $2x + 3y = 6$
has
(i) a unique solution (ii) infinitely many solutions or (iii) no solution
3. Solve the following pair of linear equations by substitution method: **[CBSE Schools 2016-2017]**
 $3x + 2y - 7 = 0,$ $2x + 3y - 3 = 0$
4. Find the value of x & y : **[CBSE Schools 2016-2017]**
 $x + 3y = 6$
 $2x - 3y = 12$
5. For which values of a and b does the following pair of linear equations have an infinite number of solutions? **[CBSE Schools 2016-2017]**
 $2x + 3y = 7$
 $(a-b)x + (a + b)y = 3a + b-2$
6. For all real values of c the pair of equations $x - 2y = 8$ and $5x - 10y = c$ have a unique solution. Justify whether this statement is true or false? **[CBSE Schools 2016-2017]**
7. For what value of k, will the following system of equations have infinite solutions? **[CBSE Schools 2016-2017]**
 $2x + 3y = 4;$ $(k + 2)x + 6y = 3k + 2.$
8. Find k so that the following pair of linear equations has no solution. **[CBSE Schools 2016-2017]**
 $3x + y = 1;$ $(2k - 1)x + (k - 1)y = 2k + 1.$
9. Solve the following pair of linear equations: **[CBSE Schools 2016-2017]**
 $2x + 3y = 6,$ $x + y = 4$
10. Solve the following pair of equations by reducing them to a pair linear equations: **[CBSE Schools 2016-2017]**
$$\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}, \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

11. Solve the following pair of equations by reducing them to a pair of linear equations:

$$\frac{5}{x-1} + \frac{1}{y-1} = 2, \frac{6}{x-1} - \frac{3}{y-2} = 1$$

[CBSE Schools 2015-2016]

12. A fraction becomes $\frac{1}{3}$ when 1st subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

[CBSE Schools 2015-2016]

13. Points A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If they travel in same direction at different speeds, they meet in 5 hours, If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

What steps do you suggest to save petrol ?

[CBSE Schools 2014-2015]

14. A man has certain notes of denomination Rs. 20 and Rs. 5 which amount to Rs. 380. If the number of notes of each kind are interchanged, they amount to Rs. 60 less than before. Find the number of notes of each denomination.

[CBSE Schools 2014-2015]

15. Mrs. Kukreti wants to make a kitchen garden behind her house in a rectangular plot by using the Bio-fertilizer. For the she done fencing by a wire 105 m around the three sides of the plot and fourth side is covered by the wall of her house. The area of the plot is increased by 50 square metres, if the length of the plot is decreased by 5 m and breadth of the plot is increased by 5 m. Find the dimensions of the plot.

What is the importance of Bio-fertilizers over Chemical fertilizer?

[CBSE Schools 2014-2015]

16. Check graphically whether the following pair of linear equations is consistent. If yes, solve it graphically:

$$2x - 5y = 0, \quad x + y = 0$$

[CBSE Schools 2014-2015]

17. Solve the following pair of equations

$$49x + 51y = 499, \quad 51x + 49y = 501$$

[CBSE Schools 2014-2015]

18. The area of a rectangle reduces by 25 sq. units, if its length is increased by 5 units and breadth is decreased by 3 units. If we increase length by 2 units and breadth by 5 units, the area increases by 285 sq. units. Find the dimensions of the rectangle.

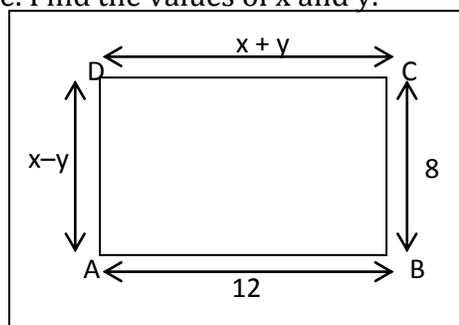
[CBSE Schools 2014-2015]

19. Without drawing the graph, find out whether the lines representing the following pair of linear equations intersect at a point or parallel or coincident; $x + 2y = 5$, $3x + 6y = 1$.

[CBSE Schools 2014-2015]

20. In given fig, ABCD is a rectangle. Find the values of x and y.

[CBSE Schools 2014-2015]



21. A person can row a boat at the rate of 5km/hour in still water. He takes thrice as much time in going 40km upstream as in going 40km downstream. Find the speed of stream. **[CBSE Schools 2014-2015]**
22. In a competitive examination, one mark is awarded for each correct answer while half mark is deducted for each wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly? **[CBSE Schools 2014-2015]**
23. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10km, the charge paid is Rs. 105 and for a journey of 15km the charge paid is Rs. 155, What are the fixed charges and the charges per km? How much does a person have to pay for travelling a distance of 25km? **[CBSE Schools 2014-2015]**
24. Draw the graphs of the equation $4x - y - 8 = 0$, $2x - 3y + 6 = 0$. Also determine the vertices of the triangle formed by the lines and X axis. **[CBSE Schools 2014-2015]**
25. A boat goes 30km upstream and 44km downstream in 10 hours. In 13 hours, it can go 40km upstream and 55km downstream. Determine the speed of the stream and that of the boat in still water. **[CBSE Schools 2014-2015]**
26. Determine graphically whether the following pair of linear equations **[CBSE Schools 2014-2015]**
 $2x + 7y = 14$, $10x + 35y = 35$ has
(i) a unique solution, (ii) infinitely many solutions or (iii) no solution
27. Determine graphically whether the following pair of linear equations **[CBSE Schools 2014-2015]**
 $3x - y = 7$, $2x + 5y + 1 = 0$ has
(i) a unique solution, (ii) infinitely many solutions or (iii) no solution
28. For Uttarakhand flood victims two sections A and B of class X contributed Rs. 1500. If the contribution of X A was Rs. 100 less than that of X B, find graphically the amounts contributed by both the sections. **[CBSE Schools 2014-2015]**

Chapter Test

Maximum Marks 30

Maximum Time: 1 hr.

- Solve :
 $2x - y - 3 = 0, 4x + y - 3 = 0$ [3]
- A and B each have certain number of oranges. A says to B, if you give me 10 of your oranges, I will have twice the number of oranges left with you. B replies, if you give me 10 of your oranges, I will have the same number of oranges as left with you. Find the number of oranges with A and B separately. [3]
- A train travels 288 km at a uniform speed. If the speed had been 4 km/h more, it would have taken 1 h less for the same journey. Find the speed of the train. [3]
- Find the value of k for which the system of equations $kx - y = 2, 6x - 2y = 3$ has (i) a unique solution (ii) no solution. Is there a value of k for which the given system has infinitely many solutions? [3]
- Solve for x and y.
 $ax + by - a + b = 0$
 $bx - ay - a - b = 0$ [3]
- Find the value of k, in which system equations $x + 3y = 2$ and $2x + ky = 8$ has no solution. [3]
- Two places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 h. If they travel towards each other, they meet in 1 h. What are the speeds of the cars? [4]
- Solve for x and y
 $\frac{15}{x-y} + \frac{22}{x+y} = 5, \quad \frac{40}{x-y} + \frac{55}{x+y} = 13$
 $x \neq y$ and $x \neq -y$ [4]
- Show graphically that each one of the following systems of equations is inconsistent (i.e., has no solution).
 $3x - 4y - 1 = 0, 2x - \frac{8}{3}y + 5 = 0$ [4]

Answers

- | | | |
|--|-----------------------|-----------------|
| 1. $x=1, y=-1$ | 2. 70,50. | 3. 32 km/hr. |
| 4. (i) $k \neq 3$ (ii) $k=3$ and $k=4$, not possible. | 5. $x = 1, y = -1$ | |
| 6. $k=6$. | 7. 60 km/h., 40 km/h. | 8. $x=8, v=3$. |

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