Polynomials

Chapter Flowchart

The Chapter Flowcharts give you the gist of the chapter flow in a single glance.

Polynomials

An algebraic expression of the form $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots\ldots a_{n-1}x + a_n$, where a_0, a_1, a_2,…….a_n are real numbers, n is non-negative integer and $a_0 \neq 0$ is called a polynomial of degree n.
Degree: The highest power of x in a polynomial p(x) is called the degree of polynomial.

Type of polynomial

(i)Constant polynomial
A polynomial of degree zero is called a constant polynomial and it is of the form p(x) = k.

(ii)Linear polynomial
A polynomial of degree one is called linear polynomial and it is of the form p(x) = ax +b where a $\neq 0$

(iii) Quadratic polynomial
A polynomial of a degree two is called quadratic polynomial and it is of the form $p(x) = ax^2 + bx + c$, where a, b, c are real number and $a \neq 0$

(iv) Cubic polynomial
A polynomial of degree three is called cubic polynomial and it is the of form $p(x) = ax^3 + bx^2 + cx + d$ where a, b, c, d are the real numbers and $a \neq o$

(v) Bi-quadratic polynomial
A polynomial of degree four is called bi-quadratic polynomial and it is of the form $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d, e are real numbers and $a \neq 0$.

Division algorithm

Division polynomial for polynomials
If p(x) and g(x) are any two polynomials with ,$g(x) \neq 0$ then we can find polynomials q(x) and r(x) such that p(x) = q(x) × g(x) +r (x),
where r(x) =0 or
degree of r(x) <degree of g(x).
Step-1 Divide the highest degree term of the dividend by the highest degree term o the divisor and obtained the remainder.
Step-2 If the remainder is 0 or degree of remainder is less than the divisor, then we cannot continue the division any further. If degree of remainder is equal to or more than divisor, then repeat step-1.

Zeroes of polynomial

α is said to be zero of a polynomial p(x) if $p(\alpha)=0$.
Properties:
(i) Geometrically, the zeroes of a polynomial p(x) are the x coordinates of the points, where the graph of y = p(x) intersects the x-axis.
(ii) A polynomial of a degree 'n' can have at most n zeroes. That is a quadratic polynomial can have at most 2 zeros and a cubic polynomial can have at most 3 zeroes.
(iii) 0 may be a zero of a polynomial.
(iv) A non-zero constant polynomial has no zeroes.

Nature of zeros of Quadratic Polynomial

Discriminant of a quadratic polynomial

For polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, the expression $b^2 - 4ac$ is known as its discriminant 'D'

$\therefore D = b^2 - 4ac$

Properties:

(i) If $D > 0$ graph of $p(x) = ax^2 + bx + c$ will intersects the x –axis at two distinct points. The x - coordinates of points of intersection with x-axis are known as 'Zeroes' of p(x).

(ii) If $D = 0$ graph of $p(x) = ax^2 + bx + c$ will touch the x-axis at exactly one point.

$\therefore$ p(x) will have only one 'Zero'.

(iii) If $D < 0$ graph of $p(x) = ax^2 + bx + c$ will neither touch nor intersect the x-axis.

$\therefore$ p(x) will not have any real 'zero'

Relationship between the zeroes and the coefficients of a polynomial:

(i) If α, β are zeroes of $p(x)\ ax^2 + bx + c$, then

Sum of zeroes = $\alpha + \beta = \frac{-b}{a} = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^2}$.

Product of zeroes = $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{Coefficient of } x^2}$.

(ii) If α, β, γ are zeroes of $p(x) = ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-\text{ Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\ \beta\gamma = \frac{-d}{a} = \frac{\text{constant term}}{\text{Coefficient of } x^3}$$

(iii)If $\alpha, \beta,$ are the roots of a quadratic polynomial p(x), then $p(x) = x^2 -$ (sum of zeroes) x+ product of zeroes. i.e. $p(x) = x^2 - \alpha + \beta\ x + \alpha\beta$

(iv) If α, β, γ are the roots of cubic polynomial p(x) , then $p(x) = x^3 -$(sum of zeroes)$x^2 -$(product of zeroes taken two at a time)x – product of zeroes

$\Rightarrow$ $p(x) = x^3 - \alpha + \beta + \gamma\ x^2 + \alpha\beta + \beta\gamma + \gamma\alpha\ x - \alpha\beta\gamma$

Graph of polynomial

(i) Graph of a linear polynomial $p(x) = ax + b$ is straight line.

(ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola, open upwards like $\cup$ if $a > 0$.

(iii) Graph of quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola, open downwards like $\cap$ if $a < 0$.

(iv) In general, a polynomial p(x) of degree n crosses the x-axis at, at most n points.

Revision Question Bank

1. Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and their coefficients.

2. Find the value of k for which a –3b is a factor of $a^4 - 7a^2b^2 + kb^4$. Hence, for this value of k, factorise $a^4 - 7a^2b^2 + kb^4$ completely.

3. Find the values of a and b for which $ax^3 - 11x^2 + ax + b$ is exactly divisible by $x^2 - 4x - 5$.

4. What must be subtracted to $f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$, so that the resulting polynomial is divisible by $g(x) = x^2 + 2x - 3$?

5. The cubic polynomial f(x) is such that the coefficient of x^3 is –1 and the zeroes of f(x) are 1,2 and k. Given that, f (x) has a remainder of 8 when divided by x – 3, find

(i) the value of k. (ii) the remainder when f(x) is divided by x + 3.

6. If one root of the polynomial $f(x) = 5x^2 + 13x + k$ is reciprocal of the other, then find the value of k.

7. If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be px + q. Then, find the values of p and q.

8. Find the value of k for which $x^2 + (k-1)x + k^2 - 16$ is exactly divisible by (x – 3) but not divisible by x+4.

9. The expressions $x^3 - ax + a^2$ and $ax^3 + x^2 - 17$ have the same remainder when divided by x – 2. Find the,

(i) possible values of a. (ii) remainder.

10. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$, then prove that

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$$

Answers

1. $\frac{2}{3}, \frac{-1}{7}$
2. k=–18, $(a + 3b)(a-3b)(a^2 + 2b^2)$
3. a =2, b = 15.
4. –61x + 65.
5. (i) k = 7, (ii) 200.
6. 5.
7. p = 2, q = 3.
8. k = – 5.
9. (i) a = 3, or a = 7. (ii) 11, 43.

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Previous years Question Bank

1. Check whether polynomial $x^2 + 2x$ is a factor of the polynomial $x^4 + 2x^3 - x^2 - 2x$. Verify by division algorithm. **[CBSE Schools 2016-17]**

2. Obtain all other zeros of the polynomial $2x^4 + 3x^3 - 15x^2 - 24x - 8$, if two of its zeroes are $2\sqrt{2}$ and $-2\sqrt{2}$. **[CBSE Schools 2016-17]**

3. Mr. Kulkarni has asked his friends to do carpooling for commuting to office because their offices are located in the same tower of the city. For this they calculated the total expense of the fuel and other charges which together is represented by $x^3 + 8x^2 + 16x + 9$. If there are $x + 1$ members, who are sharing, find the share of each member.

Why Mr. Kulkarni has taken such initiative? **[CBSE Schools 2016-17]**

4. If $x^3 - 6x^2 + 6x + k$ is completely divisible by $x - 3$, then find the value of k. **[CBSE Schools 2016-17]**

5. If two zeroes of a polynomial $x^3 - 3x^2 + 2$ are $1 + \sqrt{3}$ and $1 - \sqrt{3}$, then find the third zero. **[CBSE Schools 2016-17]**

6. What should be added in the polynomial $x^3 + 2x^2 - 9x + 1$ so that it is completely divisible by $x + 4$. **[CBSE Schools 2016-17]**

7. For what value of k, – 7 is the zero of the polynomial $2x^2 + 11x + (6k - 3)$? Also, find the other zero of the polynomial. **[CBSE Schools 2016-17]**

8. Find all other zeroes of the polynomial $x^4 - 2x^3 - 7x^2 + 8x + 12$, if two of its zeroes are – 1 and 2. **[CBSE Schools 2016-17]**

9. Using division algorithm, find the quotient and remainder on dividing f(x) by g(x) where $f(x) = 6x^3 + 13x^2 + x - 2$ and $g(x) = 2x + 1$ **[CBSE Schools 2016-17]**

10. Find a quadratic polynomial, the sum and product of whose zeroes are – 7 and – 18 respectively. Hence find the zeroes. **[CBSE Schools 2016-17]**

11. Divide the polynomial $p(x) - 2x^4 - 4x^3 - 4x^2 + 6x - 2$ by the polynomial $g(x) = x^2 - 2$ and find the quotient and the remainder. Also verify the division algorithm. **[CBSE Schools 2016-17]**

12. If one zero of the polynomial $(a + 5)$, $x^2 + 13x + 6a$ is reciprocal of the other, find the value of a. **[CBSE Schools 2016-17]**

13. What must be subtracted from the polynomial $x^4 - 4x^3 - 39x^2 - 46x - 2$ so that the resulting polynomial is exactly divisible by $x^2 - 5x + 6$. **[CBSE Schools 2016-17]**

14. Show that 2, – 1 and $\frac{1}{2}$ are the zeroes of the cubic polynomial $f(x) = 2x^3 - 3x^2 - 3x + 2$ and then verify that the sum of zeroes = $\frac{-\text{coeff of } x^2}{\text{coeff of } x^3}$ **[CBSE Schools 2016-17]**

15. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficient. **[CBSE Schools 2015-16]**

16. If α and β are the zeroes of the polynomial $4x^2 + 3x + 7$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. **[CBSE Schools 2015-16]**

17. Find the quadratic polynomial whose zeroes are 3 and –4 respectively. **[CBSE Schools 2015-16]**

18. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x). **[CBSE Schools 2015-16]**

19. Find the values of a and b so that $x^4 + x^3 + 8x^2 + ax - b$ is divisible by $x^2 + 1$. **[CBSE Schools 2015-16]**

20. Find the quadratic polynomial whose zeroes are $\sqrt{3} + \sqrt{5}$ and $\sqrt{3} - \sqrt{5}$. **[CBSE Schools 2014-15]**

21. Government of Delhi allotted Relief Fund to help the families whose houses and shops were burned in a fire accident. The fund is represented by $6x^3 - 11x^2 + 15x - 24$. The fund is equally divided between each of the families of that accident. Each family receives an amount of $3x - 7$. After distribution, $7x + 11$ amount is left. The District Magistrate decided to use this amount to develop the infrastructure of the area. Find the number of families which received relief fund from Government.
What value has been depicted here? **[CBSE Schools 2014-15]**

22. Find all the zeroes of the polynomial $8x^4 + 8x^3 - 18x^2 - 20x - 5$, if it is given that two of its zeroes are $\sqrt{\frac{5}{2}}$ and $-\sqrt{\frac{5}{2}}$. **[CBSE Schools 2014-15]**

23. For what value of k, (–4) is zero of polynomial $x^2 - x - (2k+2)$? **[CBSE Schools 2014-15]**

24. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out be $(ax + b)$, find a and b. **[CBSE Schools 2014-15]**

25. If a, p are zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are (2a) and (2p). **[CBSE Schools 2014-15]**

26. If the polymomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b. **[CBSE Schools 2014-15]**

27. If one of the zero of the quadratic polynomial $x^2 + 3x + k$ is 2, find the value of k. **[CBSE Schools 2014-15]**

28. If α and β are the zeros of the polynomials $f(x) = x^2 - 2x + 5$, then find the quadratic polynomial whose zeros are $\alpha + \beta$ and $\frac{1}{\alpha} + \frac{1}{\beta}$. **[CBSE Schools 2014-15]**

29. If α and β are the zeros of a polynomial $4x^2 + 5x + 4$, then find the value of $\alpha + \beta + 2\alpha\beta$ **[CBSE Schools 2014-15]**

30. Quadratic polynomial $4x^2 + 12x + 9$ has zeros as α and β. Now form a quadratic polynomial whose zeros are $\alpha - 1$ and $\beta - 1$. **[CBSE Schools 2014-15]**

Chapter Test

Maximum Marks: 30 **Maximum Time: 1 hr.**

1. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $(a - b)$, a and $(a + b)$, then find the values of a and b. **[3]**

2. Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$ and verify the relation between the coefficients and the zeroes of the above polynomial. **[3]**

3. If 2 and 3 are the zeroes of polynomial $3x^2 - 2kx + 2m$, find the values of k and m. **[3]**

4. α and β are zeroes of the polynomial $x^2 - 6x + a$,find the value of a, if $3\alpha + 2\beta = 20$. **[3]**

5. If the product of two zeros of the polynomial $f(x) = 2x^3 + 6x^2 - 4x + 9$ is 3. Find its third zero. **[3]**

6. If one zero of the polynomial $f(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other. Then find the value of k. **[3]**

7. If α and β are the two zeroes of the polynomial $21y^2 - y - 2$, then find a quadratic polynomial whose zeroes are 2α and 2β. **[4]**

8. The sum of remainder obtained when $x^3 + (k + 4)x + k$ is divided by $x - 2$ and when it is divided by $x + 3$ is 0. Find the value of k. **[4]**

9. Find all the zeroes $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$. **[4]**

Answers

1. $a = 1, b = \pm\sqrt{2}$.

2. $\frac{4}{3}, -\frac{3}{2}$.

3. $k = \frac{15}{2}, m = 9$.

4. –16.

5. $-\frac{3}{2}$

6. $k = 2$

7. $21x^2 - 2x - 8$.

8. $k = 23$.

9. 1 and $-\frac{1}{2}$

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