Real numbers
Chapter Flowchart

The Chapter Flowcharts give you the gist of the chapter flow in a single glance.

Real Numbers
A collection of rational and irrational numbers.

Properties of real numbers

Euclid's division lemma
Given positive integers a and b, there exist unique integers q and r satisfying
$a = bq + r$, $0 \leq r < b$

Euclid's Division algorithm

This is base on Euclid's division lemma. According to this, the HCF of any two positive integers a and b , with a >b , is obtained as follows:
Step-1 Applying the division lemma to find q and r, where $a = bq + r$, $0 \leq r < b$.

Step-2 If r = 0, the HCF is b, If $r \neq 0$, then apply Euclid's lemma to b and r.
Step-3 Continue the process till the remainder is zero. The divisor at this stage or the non – zero remainder at the previous stage is the HCF of a and b.

Fundamental theorem of arithmetic
Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occurs.

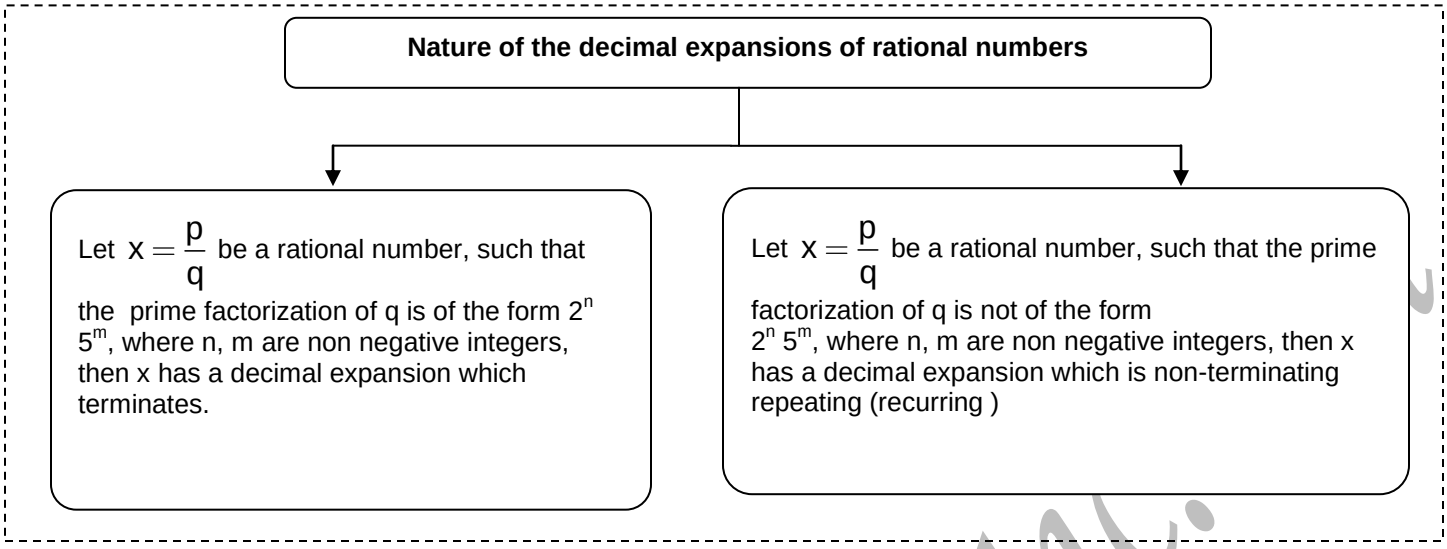
If p is a prime and p divides a^2, then p divides a, where a is a positive integer.

If x is any rational number whose decimal expansion terminates, then we can express x in the form $\frac{p}{q}$, where p and q are coprime, and the prime factorization of q is of the form $2^m 5^n$ where m, n are non- negative integers.

- For any two positive integers a and b,
  HCF (a, b) × LCM (a, b) = a × b.
- For any three positive integers a, b and c

$$\text{LCM}(a,b,c) = \frac{a\,b\,c\ \ \text{HCF}(a,b,c)}{\text{HCF}(a,b)\ .\ \text{HCF}(b,c)\ .\ \text{HCF}(c,a)}$$

$$\text{HCF}(a,b,c) = \frac{a\,b\,c\ \ \text{LCM}(a,b,c)}{\text{LCM}(a,b)\ .\ \text{LCM}(b,c)\ .\ \text{LCM}(c,a)}$$



Revision Question Bank

1. Find the HCF of 30 and 70 by using factor tree method.

2. Prove that $5-2\sqrt{3}$ is an irrational number.

3. Find the HCF of 81 and 237 and express it as a linear combination of 81 and 237.

4. Find the HCF of 65 and 117 and express it in the form 65m +117n.

5. Prove that one of the even three consecutive positive integers is divisible by 3.

6. Prove that, if x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

7. Prove that the square of any positive integer is of the form 3m or 3m +1 but not of the form 3m+2.

8. A sweet seller has 420 kaju barfis and 130 badam barfis, he wants to stack them in such a way that each stack has the same number and then take up the least area of the tray. What is the number of barfis that can be placed in each stack for this purpose?

9. Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic-wise and the height of each stack is same. The number of English books is 96, the number of Hindi books is 240 and the number of Mathematics books is 336. Assuming that the books are of same thickness. Determine the number of stacks of English, Hindi and Mathematics books.

10. State whether $1.456 + \frac{1}{9}$ is a rational number.

Answers

1. 10.
3. 3, 3 = 237×81 y, where x = 13, y = –38.
4. m = 2, n = 1.
8. 10.
9. 48.
10. Rational

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Previous years Question Bank

1. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number. **[CBSE Schools 2016-17]**

2. Prove that $n^3 - n$ is divisible by 6 for any positive integer n. **[CBSE Schools 2015-16]**

3. Can the number 6^n, n being a natural number, end with the digit 5? Give reasons. **[CBSE Schools 2016-17]**

4. Find the prime factorisation of the denominator of the rational number equivalent to 8.39. **[CBSE Schools 2016-17]**

5. Find LCM of 92 and 510. Also find their HCF by using LCM. **[CBSE Schools 2016-17]**

6. The product of two numbers x and y is 217728. Find the LCM and HCF of x and y if it is giver that LCM (x, y) = 42. HCF(x, y). **[CBSE Schools 2016-17]**

7. Find the HCF of 1620, 1725 and 255 by Euclid's Division algorithm. **[CBSE Schools 2015,17]**

8. Prove that $\sqrt{3}$ is an irrational number. Hence show that $7 + 2\sqrt{3}$ is also an irrational number. **[CBSE Schools 2015,17]**

9. State Euclid division lemma. If Euclid lemma is used for a < b as a = b q + r, then which of a, b, q, or r is necessarily zero. **[CBSE Schools 2016-17]**

10. Prove that the sum of a rational number and an irrational number is always irrational. **[CBSE Schools 2016-17]**

11. Dhudnath has two vessels containing 720 ml and 405 ml of milk respectively. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled. **[CBSE Schools 2016-17]**

12. Find LCM and HCF of 18, 24 an 54 by prime factorisation method. **[CBSE Schools 2016-17]**

13. Use Euclid division algorithm to find that the pair of numbers 615, 154 is co - prime or not? **[CBSE Schools 2016-17]**

14. Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pen and notepads. **[CBSE Schools 2016-17]**

15. Three tankers contain 187l, 231l and 275l of petrol respectively. Using Euclid's division algorithm find the capacity of the largest container that can measure the petrol of the three containers exact number of times. **[CBSE Schools 2016-17]**

16. An army contingent of 678 soldiers is to march behind an army band of 36 members in a Republic Day parade. The two groups are to march in the same number of columns. What is the maximum number of columns they can march? **[CBSE Schools 2016-17]**

17. Find HCF of 378,180 and 420 by prime factorization method. Is HCF $\times$ LCM of three numbers equal to the product of the three numbers? **[CBSE Schools 2016-17]**

18. State whether the real number 52.0521 is rational or not. If it is rational express it in the form $\frac{p}{q}$, where p, q are co-prime, integers, and $q \neq 0$. What can you say about prime factorization of q? **[CBSE Schools 2016-17]**

19. What is the HCF and LCM of two prime numbers a and b? **[CBSE Schools 2016-17]**

20. Three alarm clocks ring at intervals of 6, 9 and 15 minutes respectively. If they start ring together, after what time will they next ring together? **[CBSE Schools 2015,16]**

21. Show that the square of any positive integer is of the form 3m or, 3m + 1 for some integer m. **[CBSE Schools 2015,16]**

22. Use Euclid's division Lemma to show that the cubes of any positive integer is either of the form 9m, 9m + 1 or, 9m + 8 for some integer m. **[CBSE Schools 2015,16]**

23. Find the HCF of numbers 72 and 96 by Euclid's division algorithm and express it in the form 96m + 72n, where m and n are integers. **[CBSE Schools 2015,16]**

24. Find HCF and LCM of 12, 63 and 99 using prime factorisation method. **[CBSE Schools 2016,17]**

25. The LCM of two numbers is 2079 and their HCF is 27. If one of the number is 297. Find the other number. **[CBSE Schools 2015-16]**

26. Three alarm clocks ring at intervals of 4, 12 and 20 minutes respectively. If they start ringing together, after how much time will they next ring together? **[CBSE Schools 2015-16]**

27. If two positive integers x and y are expressible in terms of primes as $x = p^2q^3$ = and $y = p^3q$, what can you say that about their LCM and HCF. Is LCM a multiple of HCF? Explain. **[CBSE Schools 2015-16]**

28. On a morning walk, three persons steps off together and their steps measure 40 cm, 42 cm, and 45 cm respectively. What is the minimum distance each should walk so that each can over same distance in complete steps? **[CBSE Schools 2015-16]**

29. Show that $\sqrt{3}+\sqrt{5}^{\,2}$ is an irrational number. **[CBSE Schools 2015-16]**

30. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal. **[CBSE Schools 2014-15]**

31. Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pen and notepads. **[CBSE Schools 2014-15]**

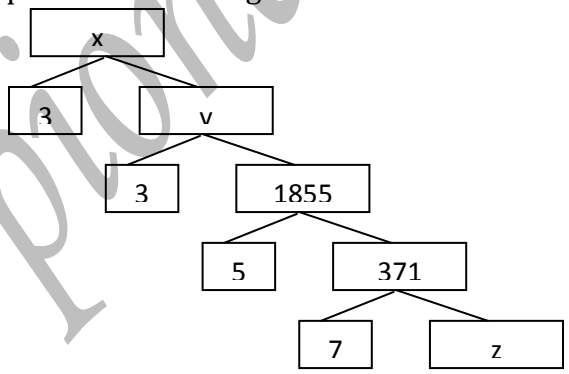
32. Find the HCF of 256 and 36 using Euclid's Division Algorithm. Also find their LCM and verify that HCF × LCM = product of the two numbers. **[CBSE Schools 2014-15]**

33. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the columns. What is the maximum number of columns in such a way that each column has equal number of members? **[CBSE Schools 2014-15]**

34. Euclid's division lemma state that for any positive integers a and b, their exist unique integers q and r such that a = bq + r where r must satisfy the condition_____. **[CBSE Schools 2014-15]**

35. Write the decimal expression $\frac{27}{1250}$ without actual division. **[CBSE Schools 2015,17]**

36. Complete the following factor tree and find the composite number x **[CBSE Schools 2014-15]**



Chapter Test

Maximum Marks: 30 **Maximum Time: 1 hour**

1. If the HCF of 85 and 153 is expressible in the form of 85m –153, then find the value of m. **[3]**

2. Given, a real number in decimal expansion is $43.\overline{123456789}$. Check whether they are rational or not. If they are rational and of the form p/q, then what can you say about the prime factors of q? **[3]**

3. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6, respectively. **[3]**

4. Show that every positive odd integer is of the form (6q + 1) or (6q + 3) or (6q + 5) for some integer q. **[3]**

5. Find the largest number which divides 248 and 1032 leaving remainder 8 in each case. **[3]**

6. Find the HCF of 231 and 396, using Euclid's division algorithm. **[3]**

7. Prove that $2\sqrt{3} - 7$ is an irrational. **[4]**

8. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or (3 m +1) for some integer m. **[4]**

9. In a seminar, the number of participants in English, Hindi and Mathematics are 60, 84 and 108, respectively.

(i) Find the minimum number of rooms required, if in each room the same number of participants are to be seated and all of them being in the same subject.

(ii) Which mathematical concept is used to solve the above question?

(iii) What values (quality) are hidden behind conducting the seminar? **[4]**

Answers

1. m =2
2. Rational, prime factors of q are not of the form 2^n5^m.
3. 63
5. 16
6. 33
9. (i) 21 (ii) Euclid's division algorithm (iii) Equality and honesty.

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