Pre-Regional Mathematical Olympiad Solution– 2017

Time: 2.5 hours. **Maximum Marks: 150**

[Each Question carries 5 marks]

1. How many positive integers less than 1000 have the property that the sum of the digits of each such number is divisible by 7 and the number itself is divisible by 3?

Ans. 28

Solution: coefficient of x^{21} : $(x^0 + x^1 + x^2 + ... + x^9)^3 = \frac{(1-x^{10})^3}{(1-x)^3}$

$(1 - 3x^{10} + 3x^{20} + ...)(1 - x)^{-3} = {}^{3+21-1}C_{21} - 3.\ {}^{3+11-1}C_{11} + 3.3$

$= {}^{23}C_{21} - 3.\ {}^{13}C_{11} + 9 = \frac{23\times 22}{2} - \frac{3\times 13\times 12}{2} + 9$

$= 23 \times 11 - 39 \times 6 + 9$

$= 253 - 234 + 9 = 28$

2. Suppose a, b are positive real numbers such that $a\sqrt{a} + b\sqrt{b} = 183$, $a\sqrt{b} + b\sqrt{a} = 182$. Find $\frac{9}{5}(a+b)$.

Ans. 73

Solution: $a\sqrt{a} + b\sqrt{b} = 183$ and $a\sqrt{b} + b\sqrt{a} = 182$ we have to find $\frac{9}{5}(a+b)$

Let $a = A^2$, $b = B^2$

$A^3 + B^3 = 183$……………… (i)

A2B + B2A = 182…………..(ii)

(1) + (2)

$A^3 + B^3 + 3AB(A + B) = 183 + 3 \times 182$

$(A + B)^3 = 183 + 546$

$(A + B)^3 = 729$

$\Rightarrow A + B = 9$

from (2) $\rightarrow$ AB (A + B) = 182

$AB = \frac{182}{9}$

$a + b = A^2 + B^2 = (A + B)^2 - 2AB$

$= 81 - \frac{364}{9} = \frac{365}{9}$

$\frac{9}{5} \times \frac{365}{9} = 73$

3. A contractor has two teams of workers : team A and team B. Team A can complete a job in 12 days and team B can do the same job in 36 days. Team A starts working on the job and team B joins team A after four days. The team A withdraws after two more days. For how many more days should team B work to complete the job?

Ans. 16

Solution: Let total worker in team A be x

Let total worker in team B be y

Given 12 x = 36 y

x = 3y

Per worker work done per day be W.

total work 36 × W × y

36Wy = 6 × Wx + MWy

M = 2 + Number of more days.

36 y = 18 y + My

M = 18y + My

M = 18

Hence 16 more days

4. Let a, b be integers such that all the roots of the equation $(x^2 + ax + 20)(x^2 + 17x + b) = 0$ are negative integers. What is the smallest possible value of a + b?

Ans. 25

Solution: $(x^2 + ax + 20)\ (x^2 + 17x + b) = 0$

so a > 0 and b > 0 since sum of roots < 0 and product > 0

(since 20 = (1 × 20) × (2 × 10) or (4 × 5)

min a = 9

$-17 = \alpha + \beta \Rightarrow (\alpha\beta) \equiv (-1,-16), (-2,-15), (-8,-9)$

min b = 16

$(a + b)_{min} = a_{min} + b_{min} = 9 + 16 = 25$

5. Let u, v, w be real numbers in geometric progression such that u > v > w. Suppose $u^{40} = v^n = w^{60}$. Find the value of n.

Ans. 48

Solution: u = a

v = ar

w = ar^2

$a^{40} = (ar)^n = (ar^2)^{60}$

$a^{20} = r^{-120} \qquad \Rightarrow a = r^{-6}$

$r^{-240} = r^{-5n} \qquad \Rightarrow \qquad 5n = 240$

$n = \frac{240}{5} = 48$

6. Let the sum $\sum_{n=1}^{9} \frac{1}{n(n+1)(n+2)}$ written in its lowest terms be $\frac{p}{q}$. Find the value of q – p.

Ans. 83

Solution: $\sum_{n=1}^{9} \frac{1}{2} \frac{n+2-n}{n(n+1)(n+2)}$

$= \frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right)$

$= \frac{1}{2}\left(\left(\frac{1}{1.2}-\frac{1}{2.3}\right)+\left(\frac{1}{2.3}-\frac{1}{3.4}\right)+........+\left(\frac{1}{9\times 10}-\frac{1}{10\times 11}\right)\right)$

$= \frac{1}{2}\left(\frac{1}{2}-\frac{1}{110}\right)$

$= \frac{1}{2}\left(\frac{55-1}{110}\right) \Rightarrow \frac{27}{110} \Rightarrow q-p=110-27=83$

7. Find the number of positive integers n such that $\sqrt{n}+\sqrt{n+1}<11$.

Ans. 29

Solution: $\sqrt{N}+\sqrt{N+1}<11$

N = 1, 2, 3, 4, 5, 6, 7, 8 .... 16 25

$\sqrt{N+1}+\sqrt{N}=11$ ...(1)

$\frac{1}{\sqrt{N+1}-\sqrt{N}}=11$

$\sqrt{N+1}-\sqrt{N}=\frac{1}{11}$ ...(2)

$2\sqrt{N+1}=\frac{122}{11} \Rightarrow \sqrt{N+1}=\frac{61}{11} \Rightarrow N+1=\frac{3721}{121}$ $N=\frac{3600}{121}=29.75$ So 29 values

8. A pen costs Rs 11 and a notebook costs Rs13. Find the number of ways in which a person can spend exactly Rs 1000 to buy pens and notebooks.

Ans. 7

Solution: Cost of a pen is Rs 11

Cost of a notebook is Rs 13

11x + 13y = 1000

$y=\frac{1000-11x}{13}\in I$

$\frac{12-11x}{13}\in I \Rightarrow \frac{(13-1)-(13x-2x)}{13}\in I$

$\frac{2x-1}{13}\in I \Rightarrow \frac{12x-6}{13}\in I \Rightarrow \frac{13x-(x+6)}{13}\in I \Rightarrow \frac{x+6}{13}\in I$

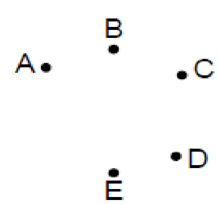
$x=13\lambda-6 \quad (1\le x\le 90)$

$x\in\{7, 20, ..., 85\}\ \{\lambda=\{1,2, ..., 3\}\}$

9. There are five cities A, B, C, D, E on a certain island. Each city is connected to every other city by road. In how many ways can a person starting from city A come back to A after visiting some cities without visiting a city more than once and without taking the same road more than once? (The order in which he visits the cities also matters. e.g., the routes $A \to B \to C \to A$ and $A \to C \to B \to A$ are different.)

Ans. 60

Solution:



ways = $^4C_2 2! + {}^4C_3 3! + {}^4C_4 4!$

= 12 + 24 + 24 = 12 + 48 = 60

10. There are eight rooms on the first floor of a hotel, with four rooms on each side of the corridor, symmetrically situated (that is each room is exactly opposite to one other room). Four guests have to be accommodated in four of the eight rooms (that is, one in each) such that no two guests are in adjacent rooms or in opposite rooms. In how many ways can the guests be accommodated?

Ans. 48

Solution:

✓ __ ✓ __

__ ✓ __ ✓

$2 \times 4! = 48$

11. Let f(x) = $\sin\frac{x}{3} + \cos\frac{3x}{10}$ for all real x. Find the least natural number n such that $f(n\pi + x)$ = f(x) for all real x.

Ans. 60

Solution:

$f(x) = \sin\frac{x}{3} + \cos\frac{3x}{10}$

period of $\sin\frac{x}{3}$ is $\frac{6\pi}{1}$ and period of $\cos\frac{3x}{10}$ is $\frac{2\pi}{3/10} = \frac{20\pi}{3}$

LCM is $\frac{60\pi}{1} \Rightarrow n = 60$

12. In a class, the total numbers of boys and girls are in the ratio 4 : 3. On one day it was found that 8 boys and 14 girls were absent from the class and that the number of boys was the square of the number of girls. What is the total number of students in the class?

Ans. 42

Solution:

Ratio is 4 : 3 therefore

Boys are 4x

Girls are 3x

given $(4x - 8) = (3x - 14)^2$

$9x^2 + 196 - 84x = 4x - 8$

$9x^2 - 88x + 204 = 0$

$x = \frac{88 \pm \sqrt{88^2 - 4 \times 9 \times 204}}{18} = \frac{2\left(22 \pm \sqrt{22^2 - 9 \times 51}\right)}{9}$

$= \frac{2(22 \pm 5)}{9} = 2 \times \frac{22}{9}$ or $2 \times \frac{17}{9}$

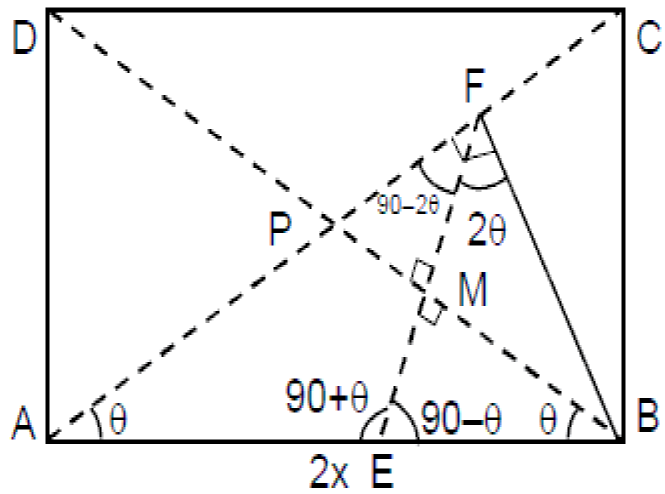
$x = 6$ or $\frac{34}{9}$

7x = 42 or non-integer $\Rightarrow$ 42 students

13. In a rectangle ABCD, E is the midpoint of AB : F is point on AC such that BF is perpendicular to AC and FE perpendicular to BD. Suppose BC = $8\sqrt{3}$. Find AB.

Ans. 24

Solution:



$\sin\theta = \frac{8\sqrt{3}}{2x} = \frac{4\sqrt{3}}{x}$

$FA = 2x\cos\theta$

$FB = 2x\sin\theta$

$MB = FB x \sin 2\theta = 2x \sin\theta \sin 2\theta$

$ME^2 + MB^2 = BE^2 = x^2$

$x^2\sin^2\theta + 4x^2\sin^2\theta\sin^2 2\theta = x^2$

$\sin^2\theta + 4\sin^2\theta\sin^2 2\theta = 1$

$4\sin^2\theta\sin^2 2\theta = \cos^2\theta$

$16\sin^4\theta = 1$

$\sin^2\theta = \frac{1}{4} \quad \Rightarrow \quad \sin\theta = \frac{1}{2}$

$\frac{4\sqrt{3}}{x} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad x = 12 \Rightarrow 2x = AB = 24$

14. Suppose x is a positive real number such that {x}, [x] and x are in the geometric progression. Find the least positive integer n such that $x^n > 100$. (Here [x] denotes the integer part of x and {x} = x – [x])

Ans. 10

Solution:

$[x]^2 = x\{x\} \longrightarrow \{x\} = a \longrightarrow [x] = ar$

$x = ar^2 \quad a + ar = ar^2$

$r^2 - r - 1 = 0$

$r = \frac{1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{1+\sqrt{5}}{2} \quad ar = I$

$a = \frac{2I}{\left(1+\sqrt{5}\right)} \quad a = \frac{I\left(\sqrt{5}-1\right)}{2} \qquad 0 < a < 1$

$0 < \frac{I\left(\sqrt{5}-1\right)}{2} < 1 \qquad 0 < I < \frac{2}{\sqrt{5}-1} \qquad 0 < I < \frac{\sqrt{5}+1}{2} \qquad I = 1$

$ar = 1 \Rightarrow a = \frac{2}{1+\sqrt{5}} = \frac{\sqrt{5}-1}{2} \qquad x = ar^2 = r = \frac{\sqrt{5}+1}{2}$

$\left(\frac{\sqrt{5}+1}{2}\right)^N > 100 \Rightarrow N\log_{10}\left(\frac{\sqrt{5}+1}{2}\right) > 2$

$N > 9.5 \Rightarrow N_{min} = 10$

15. Integers 1,2,3……..n where n > 2, are written on a board. Two numbers m, k such that 1 < m < n. 1 < k < n are removed and the average of the remaining numbers is found to be 17. What is the maximum sum of the two removed numbers?

Ans. 51

Solution: $\dfrac{\dfrac{n(n+1)}{2}-(2n-1)}{n-2}<17<\dfrac{\dfrac{n(n+1)}{2}-3}{n-2}$

$\dfrac{n^2+n-4n+2}{2(n-2)}<17<\dfrac{n^2+n-6}{2(n-2)}$

$\dfrac{n^2+3n+2}{2(n-2)}<17<\dfrac{(n+3)(n-2)}{2(n-2)}$

$\dfrac{n-1}{2}<17<\dfrac{n+3}{2}$

n < 35 and n > 31

n = 32, 33, 34

case – 1, n = 32

$\dfrac{\dfrac{n(n+1)}{2}-p}{(n-2)}=17 \Rightarrow \dfrac{n(n+1)}{2}-17(n-2)=p$

p = 18

case-2, n = 33 $\Rightarrow$ p = 34

case-3, n = 34 $\Rightarrow$ p = 51

Maximum sum = 51

16. Five distinct 2-digit numbers are in a geometric progression. Find the middle term.

Ans. 36

Let the numbers be a ar ar^2 ar^3 ar^4

since all are 2 digit number $r=\dfrac{2}{3}$ or $\dfrac{3}{2}$ (as fourth power of integers greater than 3 are 3 digit numbers)

Hence the five numbers are (16, 24, 36, 54, 81)

Hence middle term is 36.

17. Suppose the altitudes of a triangle are 10, 12 and 15. What is its semi-perimeter?

Ans. Bonus

Solution:

$h_a : h_b : h_c = 10 : 12 : 15$

$a, b, c = \dfrac{1}{10} : \dfrac{1}{12} : \dfrac{1}{15}$

a : b : c = 6 : 5 : 4

(a, b, c) = (6k, 5k, 4k)

2s = 15k

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$$\Delta = \sqrt{\frac{15k}{2}\left(\frac{15k}{2}-6k\right)\left(\frac{15k}{2}-5k\right)\left(\frac{15k}{2}-4k\right)}$$

$$\Delta = \sqrt{\frac{15k}{2}.\frac{3k}{2}.\frac{5k}{2}.\frac{7k}{2}}$$

$$\Delta = \frac{1}{4}\sqrt{15^2.7k^2} = \frac{15}{4}\sqrt{7}k^2$$

$$h_a = 10 = \frac{2\times\frac{15}{4}\sqrt{7}k^2}{6k}$$

$$60k = \frac{15}{2}\sqrt{7}k^2$$

$$8 = \sqrt{7}k \Rightarrow k = \frac{8}{\sqrt{7}}$$

$$s = \frac{15}{2}\times\frac{8}{\sqrt{7}} = \frac{60}{\sqrt{7}}$$

18. If the real numbers x, y, z are such that $x^2 + 4y^2 + 16z^2 = 48$ and $xy + 4yz + 2zx = 24$. What is the value of $x^2 + y^2 + z^2$?

Ans. 21

Solution:

$x^2 + 4y^2 + 16z^2 = 48$

$(x)^2 + (2y)^2 + (4z)^2 = 48$

$2xy + 8yz + 4zx = 48$

now we can say that

$(x)^2 + (2y)^2 + (4z)^2 - (2xy) - (8yz) - (4zx) = 9$

$[(x - 2y)^2 + (2y - 4z)^2 + (x - 4y)^2)] = 0$

$x = 2y = 4z \Rightarrow \frac{x}{4} = \frac{y}{2} = z$

$(x, y, z) = (4\lambda, 2\lambda, \lambda)$

$x^2 + 4y^2 + 16z^2 = 48$

$16\lambda^2 + 16\lambda^2 + 16\lambda^2 = 48$

so $\lambda^2 = 1$

$x^2 + y^2 + z^2 = 21\lambda^2 = 21$

19. Suppose 1, 2, 3 are the roots of the equation $x^4 + ax^2 + bx = c$. Find the value of c.

Ans. 36

Solution: 1, 2, 3 are roots of $x^4 + ax^2 + bx - c = 0$

since sum of roots is zero and fourth root is -6

Hence c = 36

20. What is the number of triples (a, b, c) of positive integers such that (i) a < b < c < 10 and (ii) a, b, c, 10 form the sides of a quadrilateral?

Ans. 73

Solution: $a + b + c > 10$

$\therefore$ (a, b, c) can be

a	b	c
1	2	8, 9
1	3	7, 8, 9
1	4	6, 7, 8, 9
1	5	6, 7, 8, 9
1	6	7, 8, 9
1	7	8, 9
1	8	9
2	3	6, 7, 8, 9
2	4	5, 6, 7, 8, 9
2	5	6, 7, 8, 9
2	6	7, 8, 9
2	7	8, 9
2	8	9
3	4	5, 6, 7, 8, 9
3	5	6, 7, 8, 9
3	6	7, 8, 9
3	7	8, 9
3	8	9
4	5	6, 7, 8, 9
4	6	7, 8, 9
4	7	8, 9
4	8	9
5	6	7, 8, 9
5	7	8, 9
5	8	9
6	7	8, 9
6	8	9
7	8	9

Total 73 cases

21. Find the number of ordered triples (a, b, c) of positive integers such that abc = 108.

Ans. 60

Solution:

$abc = 3^3 . 2^2$

$a = 3^{\alpha_1} . 2^{\beta_1}$, $b = 3^{\alpha_2} . 2^{\beta_2}$, $c = 3^{\alpha_3} . 2^{\beta_3}$

$\alpha_1 + \alpha_2 + \alpha_3 = 3$ and $\beta_1 + \beta_2 + \beta_3 = 3$

${}^5C_2$ and ${}^4C_2$

Total = ${}^4C_2 \times {}^5C_2 = 10 \times 6 = 60$

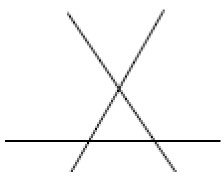
22. Suppose in the plane 10 pair wise nonparallel lines intersect one another. What is the maximum possible number of polygons (with finite areas) that can be formed?

Ans. 46

Solution:

Number of non-overlapping polygons = 56 – 20 = 46

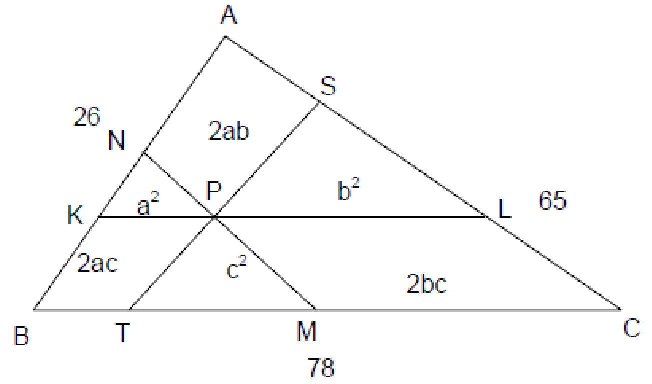
1 line divide plane into 2 regions
2 lines divide plane into 4 regions
3 lines divide plane into 7 regions
4 lines divide plane into 11 regions
5 lines divide plane into 16 regions
6 lines divide plane into 22 regions
7 lines divide plane into 29 regions
8 lines divide plane into 37 regions
9 lines divide plane into 46 regions
10 lines divide plane into 56 regions
Now open regions for 3 lines are 6



Similarly for 10 lines are 20
Note : If we consider overlapping polygons then maximum possible number of polygons = $^{10}C_3 + {}^{10}C_4 + ... + {}^{10}C_{10} = 210 - 1 - 10 - 45 = 968$

23. Suppose an integer r, a natural number n and a prime number p satisfy the equation $7x^2 - 44x + 12 = p^n$. Find the largest value of p.
Ans. 47
Solution:
$7x^2 - 44x + 12 = p^n$
$7x^2 - 42x - 2x + 12 + p^n$
$(7x - 2)(x - 6) = p^n$
$7x - 2 = p^\alpha$ and $x - 6 = p^\beta$
$(7x-2) - 7(x-6) = p^\alpha - 7p^\beta$
$40 = p^\alpha - 7p^\beta$
If $\alpha, \beta \in N$, p is divisors of 40 $\Rightarrow$ p = 2 or 5
If p = 2, $40 = 2^\alpha - 7.2^\beta \Rightarrow 2^3.5 = 2^\alpha - 7.2^\beta$
$\Rightarrow \beta = 3$ and $2^\alpha = 40 + 56 \Rightarrow \alpha \notin Z$ hence not possible
If p = 5 then $40 = 5^\alpha - 7.5^\beta \Rightarrow 2^3.5 = 5^\alpha - 7.5^\beta$
$\Rightarrow \beta = 1$ and $5^\alpha = 40 + 35 \Rightarrow \alpha \notin Z$ hence not possible
so $\beta = 0 \Rightarrow p^\alpha = 47 \Rightarrow p = 47$ and $\alpha = 1$

24. Let P be an interior point of a triangle ABC whose side lengths are 26, 65, 78. The line through P parallel to BC meets AB in K and AC in L. The line through P parallel to CA meets BC in M and BA in N. The line through P parallel to AB meets CA in S and CB in T. If KL, MN, ST are of equal lengths, find this common length.
Ans. Bonus
Solution: Let MN = ST = KL = ℓ



$$\frac{\ell}{26}=\sqrt{\frac{(\ldots+c)^2}{(a+b+c)^2}}$$

$$\frac{\ell}{26}=\frac{c}{a+b+c}$$

$$\frac{\ell}{65}=\frac{c}{a+b+c}$$

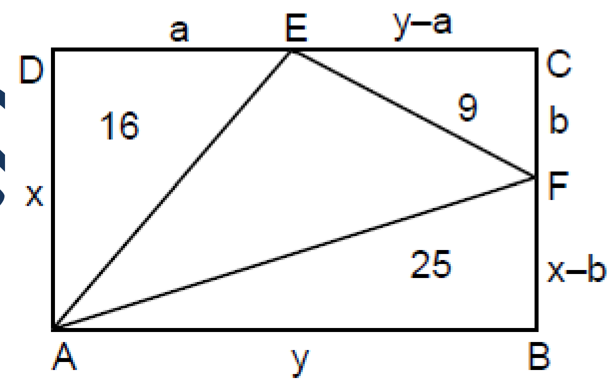
$$\frac{\ell}{78}=\frac{\ell}{26}=\angle$$

$$\frac{\ell}{26}+\frac{\ell}{65}+\frac{\ell}{78}=\angle \quad \ell$$ which is not possible as ℓ has to be less than 26

25. Let ABCD be a rectangle and let E and F be points on CD and BC respectively such that area (ADE) = 16, area (CEF) = 9 and area (ABF) = 25. What is the area of triangle AEF?

Ans. 30

Solution:



$xa = 32 \Rightarrow xa = 32$

$b(y - a) = 18 \Rightarrow by - ab = 18$

$y(x - b) = 50 \Rightarrow xy - by = 50$

$by - \frac{32b}{x} = 18 \Rightarrow b = \frac{18b}{xy-32}$

$xy - \frac{18xy}{xy-32} = 50 \qquad xy = t$

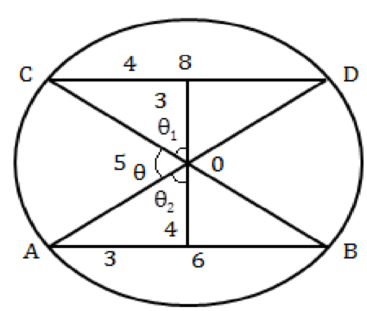
$\frac{t^2-32t-18t}{t-32} = 50 \qquad t^2 - 50t = 50t - 1600 \qquad t^2 - 100t + 1600 = 0$

$t = 80, 20 \qquad$ Now $xy = 80 \qquad$ Area of $\Delta AEF = 80 - (16 + 9 + 25) = 30$

26. Let AB and CD be two parallel chords in a circle with radius 5 such that the centre O lies between these chords. Suppose AB = 6, CD = 8. Suppose further that the area of the part of the circle lying between the chords AB and CD is $(m\pi + n) / k$, where m, n, k are positive integers with gcd(m, n, k) = 1. What is the value of m + n + k?

Ans. 75

Solution:



$$A = 2\left[\frac{1}{2}\times 25\times\theta\right]+\frac{1}{2}\times 3\times 8+\frac{1}{2}\times 4\times 6$$

$$\theta = [\pi - (\theta_1+\theta_2)] = \left[\pi - \left(\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{3}{4}\right)\right]$$

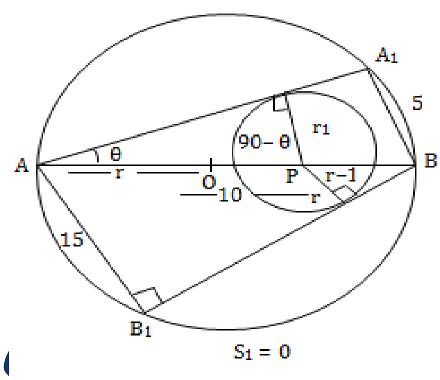
$$\theta = \frac{\pi}{2} \qquad A = 24 + \frac{25\pi}{2} \Rightarrow A = \frac{48+25\pi}{2}$$

$$(m+n+k) = (48+2+25) = 75$$

27. Let Ω_1 be a circle with centre O and let AB be diameter of Ω_1. Let P be a point on the segment OB different from O. Suppose another circle Ω_2 with centre P lies in the interior of Ω_1. Tangents are drawn from A and B to the circle Ω_2 intersecting again at A_1 and B_1 respectively such that A_1 and B_1 are on the opposite sides of AB. Given that $A_1B = 5$, $AB_1 = 15$ and $OP = 10$, find the radius of Ω_1.

Ans.20

Solution:



$$\frac{r_1}{r+10} = \frac{5}{2r} \quad .....(1)$$

$$\frac{r_1}{r-10} = \frac{15}{2r} \quad ......(2)$$

$$\frac{r-10}{r+10} = \frac{1}{3}$$

$3r - 30 = r + 10$

$2r = 40$

$r = 20$

28. Let p, q be prime numbers such that $n^{3pq} - n$ is a multiple of 3pq for all positive integers n. Find the least possible value of p + q.

Ans. 28

29. For each positive integer n, consider the highest common factor h_n of the two numbers n! + 1 and (n + 1)!. For n < 100, find the largest value of h_n.

Ans. 97

Solution:

n! + 1 is not divisible by 1, 2, ……, n

(n + 1)!. Is divisible by 1, 2, …., n

so HCF $\geq$ n + 1

also (n + 1) ! is not divisible by n + 2, n + 3 ………

so HCF can be n + 1 only

Let us start by taking n = 99

$\Rightarrow$ 99! + 1 and 100!

HCF = 100 is not possible as 100 divides 99!

composite number will not be able to make it

so let us take prime i.e. n = 97

now 96! + 1 and 97! are both divisible by 97

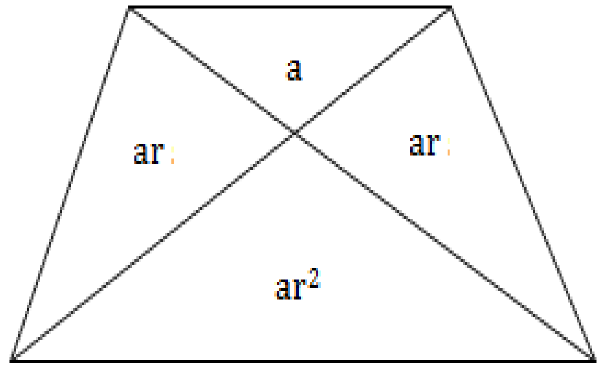
so HCF = 97

(by Wilson's theorem (p – 1)! + 1) is divisible by pp)

30. Consider the areas of the four triangles obtained by drawing the diagonals AC and BD of a trapezium ABCD. The product of these areas, taken two at time, are computed. If among the six products so obtained, two product are 1296 and 576, determine the square root of the maximum possible area of the trapezium to the nearest integer.

Ans. 13

Solution:



Case – 1 $a^2r^2 = 1296$

$ar^2 = 576$

$$r = \left(\frac{36}{24}\right)^2 = \frac{9}{4}$$

$$a^2 = 24 \times 24 \times \frac{4}{9} \Rightarrow a = 24 \times \frac{2}{3}$$ **a = 16**

Case-2 $a^2r = 576$

$a^2r^3 = 1296$

$$r^2 = \frac{1296}{576}$$

$$r^2 = \left(\frac{3}{2}\right)^2 \Rightarrow r = \frac{3}{2}$$

$$a^2 = 576 \times \frac{2}{3} \qquad a^2 = 192 \times 2 \qquad a^2 = 384$$

Case-3 $a^2r^3 = 1296$

$a^2r^2 = 576$

$$r^2 = \frac{1296}{576}$$

$$r = \frac{9}{4} \qquad a^2 = \frac{576 \times 16}{81} \qquad a = \frac{32}{3}$$ area = $a(r + 1)^2$

Case-1 : area = $16\left(1 + \frac{9}{4}\right)^2 = 169 \Rightarrow$ square rot is 13

Case-2: area = $\left(1 + \frac{3}{2}\right)^2 = 122.47$

Case-3: area = $\frac{32}{3}\left(1 + \frac{9}{4}\right)^2$ so maximum area is 13