

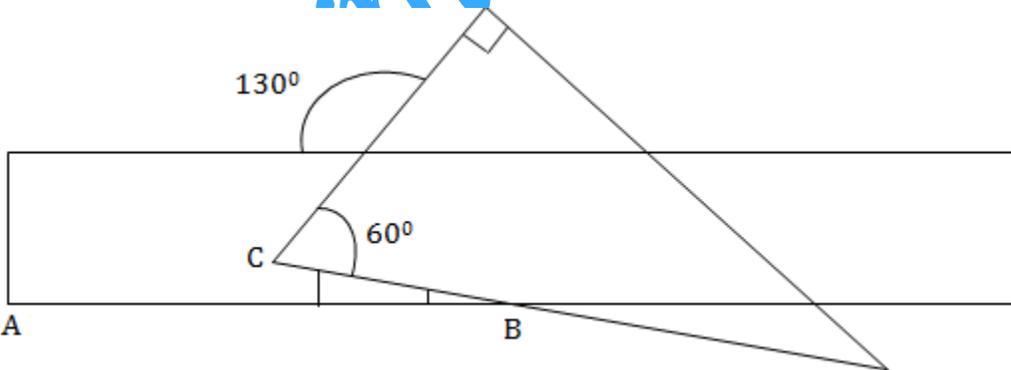
Regional Mathematical Olympiad 2016 First round

Time: 3 hours

Total Marks: 100

Note: Attempt all the questions

1. Suppose in a given collection of 2016 integer, the sum of any 1008 integers is positive. Show that sum of all 2016 integers is positive. [10]
2. On a stormy night ten guests came to dinner party and left their shoes outside the room in order to keep the carpet clean. After the dinner there was a blackout, and the guests leaving one by one, put on at random, any pair of shoes big enough for their feet. (Each pair of shoes stays together). Any guest who could not find a pair big enough spent the night there. What is the largest number of guests who might have had to spend the night there? [15]
3. a, b, c, d are integers such that $ad + bc$ divides each of a, b, c and d . Prove that $ad + bc = \pm 1$. [15]
4. Prove that $(4\cos^2 9^\circ - 3)(4\cos^2 27^\circ - 3) = \tan 9^\circ$. [10]
5. Given a rectangle $ABCD$, determine two points K and L on the sides BC and CD such that the triangles ABK, AKL and ADL have same area. [15]
6. Positive integers a, b, c satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} < 1$. Prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{41}{42}$. Also prove that equality in fact holds in the second inequality. [10]
7. Two of the Geometry box tools are placed on the table as shown. Determine the angle $\angle ABC$. [10]



8. At some integer points a polynomial with integer coefficients take values 1, 2 and 3. Prove that there exist not more than one integer at which the polynomial is equal to 5. [15]