

JEE (Advanced)-2016 (Paper-2)

PART I: PHYSICS

SECTION:2 (Maximum marks : 9)

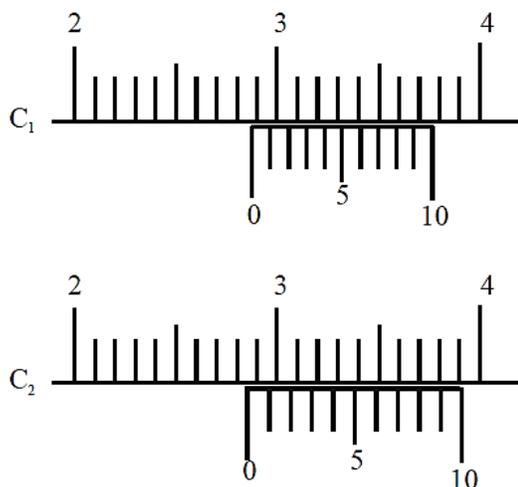
Maximum Time:

Maximum Marks: 61

- This section contains **SIX** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: -1	In all other cases.

1. There are two vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers C_1 and C_2 respectively, are



- (A) 2.87 and 2.86 (B) 2.85 and 2.82 (C) 2.87 and 2.87 (D) 2.85 and 2.83
2. The ends Q and K or two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C . Now the end P is maintained at 10°C , while the end S is heated and maintained at 400°C . The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5} \text{ K}^{-1}$, the change in length of the wire PQ is
- (A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

3. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m³ changes to a final state at $P_f = \left(\frac{1}{32}\right) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m³ in an adiabatic quasi-static process, such that $P^3V^5 = \text{constant}$. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumes V_f . The amount of heat supplied to the system in the two step process is approximately
- (A) 112 J (B) 294 J (C) 588 J (d) 813 J

SECTION:2 (Maximum marks : 12)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :
 Full Marks : +4 If only the bubble(s) corresponding to the correct option(s) is (are) darkened.
 Partial Marks : +1 For darkening a bubble corresponding to **each correct option**, Provided NO incorrect option is darkened.
 Zero Marks : 0 If none of the bubbles is darkened.
 Negative Marks : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

4. In an experiment to determine the acceleration due to gravity g , the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$. The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57s, 0.54s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement (s) is (are) true?
- (A) The error in the measurement of r is 10%
- (B) The error in the measurement of T is 3.57 %
- (C) The error in the measurement of T is 2%
- (D) The error in the determined value of g is 11%

5. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude. A about an equilibrium position x_0 . Consider two cases : (i) when the block is at x_0 ; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass $m (< M)$ is softly placed on the block after which they stick to each other. Which of the following statement (s) is (are) true about the motion after the mass m is placed on the mass M ?

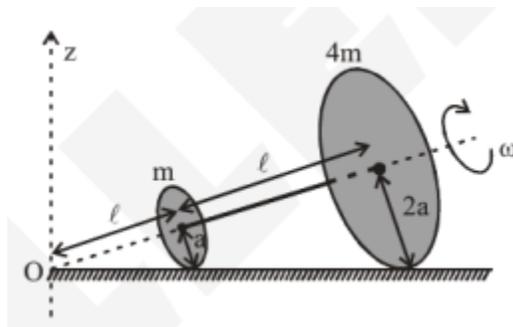
(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

6. Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by massless, right rod of length ℓ through their center. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω . The angular momentum of the entire assembly about the point 'O' is \vec{L} (see the figure). Which of the following statement (s) is (are) true?



- (A) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$
- (B) The magnitude of the z -component of \vec{L} is $55 ma^2\omega$
- (C) The magnitude of angular momentum of center of mass of the assembly about the point O is $81 ma^2\omega$
- (D) The center of mass of the assembly rotates about the z -axis with an angular speed of $\omega/5$

Section-3 (Maximum Marks:6)

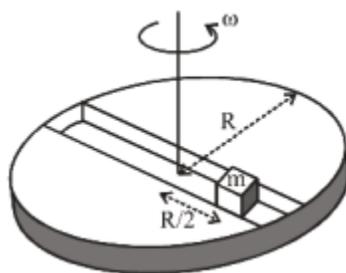
- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :
 Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
 Zero Marks : 0 In all other cases.

PARAGRAPH

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force \vec{F}_{rot} experienced by a particle of mass m moving on the rotating disc and the force \vec{F}_{in} experienced by the particle in an inertial frame of reference is

$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$, where \vec{v}_{rot} is the velocity of the particle in the rotating frame of reference and \vec{r} is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis ($\vec{\omega} = \omega \hat{k}$). A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ and is constrained to move only along the slot.



7. The distance r of the block at time t is :

- (A) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (B) $\frac{R}{2} \cos 2\omega t$ (C) $\frac{R}{2} \cos \omega t$ (D) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

8. The net reaction of the disc on the block is :

- (A) $-m\omega^2 R \cos \omega t, \dots$ (B) $m\omega^2 R \sin \omega t, \dots$
 (C) $\frac{1}{2} m\omega^2 R (e^{\omega t} - e^{-\omega t}), \dots$ (D) $\frac{1}{2} m\omega^2 R (e^{2\omega t} - e^{-2\omega t}), \dots$

Part-II Chemistry**Section-2 (Mark- 4)**

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :
Full Marks : +4 If only the bubble(s) corresponding to the correct option(s) is (are) darkened.
Partial Marks : +1 For darkening a bubble corresponding to **each correct option**, Provided NO incorrect option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

9. According to Molecular Orbital Theory,
- (A) C_2^{2-} is expected to be diamagnetic
 - (B) O_2^{2+} is expected to have a longer bond length than O_2
 - (C) N_2^+ and N_2^- have the same bond order
 - (D) He_2^+ has the same energy as two isolated He atoms

SECTION-3 : (Maximum Marks : 6)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :
 Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
 Zero Marks : 0 In all other cases.

PARAGRAPH

Thermal decomposition of gaseous X_2 to gaseous X at 298 K takes place according to the following equation: $X_2(g) \rightleftharpoons 2X(g)$

The standard reaction Gibbs energy, $\Delta_r G^\circ$, of this reaction is positive. At the start of the reaction, there is one mole of X_2 and no X . As the reaction proceeds, the number of moles of X formed is given by β . Thus, $\beta_{\text{equilibrium}}$ is the number of moles of X formed at equilibrium. The reaction is carried out at a constant total pressure of 2 bar. Consider the gases to behave ideally.

(Given : $R = 0.083 \text{ L bar K}^{-1} \text{ mol}^{-1}$)

10. The equilibrium constant K_p for this reaction at 298 K, in terms of $\beta_{\text{equilibrium}}$, is

(A) $\frac{8\beta^2}{2 - \beta_{\text{equilibrium}}}$ (B) $\frac{8\beta_{\text{equilibrium}}^2}{4 - \beta_{\text{equilibrium}}^2}$ (C) $\frac{4\beta_{\text{equilibrium}}^2}{2 - \beta_{\text{equilibrium}}}$ (D) $\frac{4\beta_{\text{equilibrium}}^2}{4 - \beta_{\text{equilibrium}}^2}$

11. The INCORRECT statement among the following, for this reaction, is

- (A) Decrease in the total pressure will result in formation of more moles of gaseous X
 (B) At the start of the reaction, dissociation of gaseous X_2 takes place spontaneously
 (C) $\beta_{\text{equilibrium}} = 0.7$
 (D) $K_c < 1$

Part III: Mathematics

Section 1 (Maximum Marks : 6)

- This section contains **Two** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLE ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+3	If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0	If none of the bubbles is darkened.
Negative Marks:		-1	In all other cases.

12. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to

- (A) $3 - \sqrt{3}$ (B) $2(3 - \sqrt{3})$ (C) $2(\sqrt{3} - 1)$ (D) $2(2 + \sqrt{3})$

13. Let $b_i > 1$ for $i = 1, 2, \dots, 101$. Suppose $\log_e b_1, \log_e b_2, \dots, \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose a_1, a_2, \dots, a_{101} are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$.

If $t = b_1 + b_2 + \dots + b_{51}$ and $s = a_1 + a_2 + \dots + a_{51}$, then

- (A) $s > t$ and $a_{101} > b_{101}$ (B) $s > t$ and $a_{101} < b_{101}$
 (C) $s < t$ and $a_{101} > b_{101}$ (D) $s < t$ and $a_{101} < b_{101}$

Section 2 (Maximum Marks: 12)

- This section contains **Three** questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- For each question, marks will be awarded in one of the following categories :
Full Marks : +4 If only the bubble(s) corresponding to the correct option(s) is (are) darkened.
Partial Marks : +1 For darkening a bubble corresponding **to each correct option**. Provided NO incorrect option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened

14. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then
- (A) $SP = 2\sqrt{5}$
- (B) $SQ:QP = (\sqrt{5} + 1):2$
- (C) the x-intercept of the normal to the parabola at P is 6
- (D) the slope of the tangent to the circle at Q is $\frac{1}{2}$
15. Let $a, b \in \mathbb{R}$ and $a^2 + b^2 \neq 0$. Suppose $S = \left\{ z \in \mathbb{C} \mid z = \frac{1}{a + ib} \right\}$ where $i = \sqrt{-1}$. If $z = x + iy$ and $z \in S$, then (x, y) lies on
- (A) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$
- (B) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$
- (C) the x-axis for $a \neq 0, b = 0$
- (D) the y-axis for $a = 0, b \neq 0$

16. Let $a, \lambda, \mu \in \mathbb{R}$. Consider the system of linear equations

$$ax + 2y = \lambda$$

$$3x - 2y = \mu$$

Which of the following statement (s) is (are) correct?

- (A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ
- (B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ
- (C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$
- (D) If $\lambda + \mu = 0$, then the system has no solution for $a = -3$

Section 3 (Maximum Marks : 6)

- This section contains **Two** questions.
- Based on each paragraph, there are **two** questions.
- Each question has Four options (A),(B),(C) and (D). Only One of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:
Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 In all other cases.

PARAGRAPH

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at points M in the first quadrant and at point N in the fourth quadrant.

17. The orthocentre of the triangle F_1MN is

- (A) $\left(-\frac{9}{10}, 0\right)$ (B) $\left(\frac{2}{3}, 0\right)$ (C) $\left(\frac{9}{10}, 0\right)$ (D) $\left(\frac{2}{3}, \sqrt{6}\right)$

18. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

- (A) 3: 4 (B) 4: 5 (C) 5: 8 (D) 2:3