Set III

1. A total of 28 handshakes were exchanged at the conclusion of a party. Assuming that each participant was equally polite toward all the other, the number of people present was :

(a) 14 (b) 28

(c) 56 (d) 8

Sol: (d)

Let No. of people is = x

and each one has to hand shake with (x - 1) persons.

$\therefore \frac{x(x-1)}{2} = 28$

$x^2 - x = 56$

$x^2 - x - 56 = 0$

$x^2 - 8x + 7x - 56 = 0$

$(x - 8)(x + 7) = 0$

$x = 8, \quad x = -7$

$\therefore$ No. of person = 8

2. In the set of equations $z^x = y^{2x}, 2^z = 2 . 4^x$, $x + y + z = 16$, the integral roots in the order

x, y, z are :

(a) 3, 4, 9 (b) 9, -5, 12

(c) 12, -5, 9 (d) 4, 3, 9

Sol:

$z^x = y^{2x}, 2^z = 2.4^x, x + y + 2 = 16.$

$2^z = 2^{1+2x}$

$\therefore z = 1 + 2x$

Only option D will satisfy. This conditions.

Options – A	$9 = 1 + 2(3)$,	$9 = 7$
Options – B	$9 = 1 + (2)(12)$,	N/equal
Options – C	$12 = 1 + 2(9)$	$12 = 19$
option – D	$9 = 1 + 2(4)$,	$9 = 9$

3. Let D represent a repeating decimal. If P denotes the r figures of D which do not repeat themselves, and Q denotes the s figures which do repeat themselves, then the incorrect expression is :

(a) D = . PQQQ ... (b) 10^r D = P.QQQ...

(c) 10^{r+s}D = PQ.QQQ... (d) $10^r (10^s - 1)$ D = Q(P – 1)

Sol: (d)

$10^r (10^s - 1)$ D = Q(P – 1)

4. A and B together can do a job in 2 days; B and C can do it four days; and A and C in $\frac{2}{5}$ days. The number of days required for A to do the job alone is :

(a) 1 (b) 3

(c) 6 (d) 12

Sol: No Answer

5. Two candles of the same height are lighted at the same time. The first is consumed in 4 hours and the second in 3 hours. Assuming that each candle burns at a constant rate, in how many hours after being lighted was the first candle twice the height of the second?

(a) $\frac{3}{4}$hr. (b) $1\frac{1}{2}$hr.

(c) 2 hr. (d) $2\frac{2}{5}$hr.

Sol: d

Let the height in each case be 1.$1-\frac{1}{4}t = 2(1 - \frac{1}{3}t)$; $\therefore t = 2\frac{2}{5}$.

6. The points of intersection of $xy = 12$ and $x^2 + y^2 = 25$ are joined in succession. The resulting figure is :

(a) a straight line
(b) an equilateral triangle
(c) a parallelogram
(d) a rectangle

Sol:

$x^2 + y^2 = 25$ in a circle with centre at the origin. $xy = 12$ is a hyperbola, symmetric with respect to the line $x = y$. $\therefore$ point of intersection are symmetric with respect to $x = y$. There are either no intersections, two intersections (if the hyperbola is tangent to the circle) or four intersections. Simultaneous solution of the equations reveals that there are four points of intersection, and that they determine a rectangle

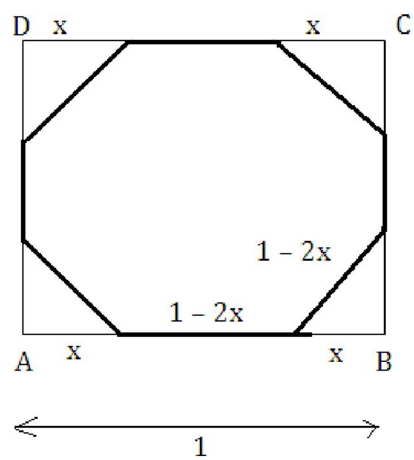
7. A regular octagon is to be formed by cutting equal isosceles right triangles from the corners of a square. If the square has sides of one unit, the leg of each of the triangles has length:

(a) $\frac{2+\sqrt{2}}{3}$

(b) $\frac{2-\sqrt{2}}{2}$

(c) $\frac{1-\sqrt{2}}{2}$

(d) $\frac{1+\sqrt{2}}{3}$



Sol:

$x^2 + x^2 = (1 - 2x)^2$

$2x^2 = 1 + 4x^2 - 4x$

$0 = 2x^2 - 4x + 1$

$$x = \frac{4 \pm \sqrt{16 - 4.2.1}}{2.2}$$

$$x = \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2}$$

$$x = \frac{2+\sqrt{2}}{2}, \frac{2-\sqrt{2}}{2}$$

8. If a and b are two unequal positive numbers; then :

(a) $\frac{2ab}{a+b} > \sqrt{ab} > \frac{a+b}{2}$

(b) $\frac{a+b}{2} > \sqrt{ab} > \frac{2ab}{a+b}$

(c) $\frac{2ab}{a+b} > \frac{a+b}{2} > \sqrt{ab}$

(d) $\frac{a+b}{2} > \frac{2ab}{a+b} > \sqrt{ab}$

Sol:

The Arithmetic Mean is (a + b)/2, the Geometric Mean is $\sqrt{ab}$, and the Harmonic Mean is 2ab/(a + b). The proper order for decreasing magnitude is (e); or Since $(a - b)^2 > 0$, we have $a^2 + b^2 > 2ab$;

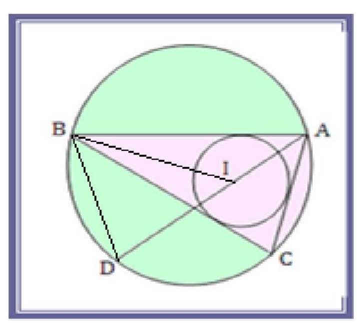
$\therefore$ $a^2 + 2ab + b^2 > 4ab$,

$a + b > 2\sqrt{ab}$ and $(a + b)/2 > \sqrt{ab}$.

Since $a^2 + 2ab + b^2 > 4ab$, we have $1 > 4ab/(a + b)^2$;

$\therefore$ $ab > 4a^2b^2/(a + b)^2$, and $\sqrt{ab} > 2ab/(a + b)$

9. In the given Figure "I" is the Incentre of ΔABC. AI when produced meets the circumcircle of ΔABC in D. If $\angle BAC = 66^0$ and $\angle ACB = 80^0$, then $\angle DBC$, $\angle IBC$ & $\angle BID$ respectively:



(a) $17^0, 33^0$ & 50^0

(b) $33^0, 50^0$ & 17^0

(c) $33^0, 17^0$ & 50^0

(d) $50^0, 33^0$ & 17^0

Sol: (c)

AD is Angle Bisector.

$\therefore \angle DBC = \angle DAC$ (Angle in the save segment)

$\therefore \angle DBC = 33^o$

$\angle A + \angle B + \angle C = 180^0$

$66^0 + \angle B + 80^o = 180^0$

$\angle B = 34^0$

$\angle IBC = \frac{1}{2}\angle B = \frac{34}{2} = 17^0$

$\angle IBC = 17^o$

$\angle BID = 50^0 + 80^0 + x = 180^0, x = 50^0$

10. You are given a sequence of 58 terms; each term has the form P + n where P stands for the product 2 . 3. 5 …….61 of all prime numbers (a prime number is a number divisible only 1 and itself) less than or equal to 61 and n takes successively the value 2, 3, 4, …., 59. Let N be the number of primes appearing in this sequence. Then N is :

(a) 0 (b) 16

(c) 17 (d) 57

Sol:a

All those numbers P + n of the sequence P + 2, P + 3, …., P + 59 for which n is prime are divisible by n because n already occurs as a factor in P. In the remaining members, n is composite and hence can be factored into primes that are smaller than 59. Hence all terms are divisible by n.

$\therefore$ all members of the sequence are composite

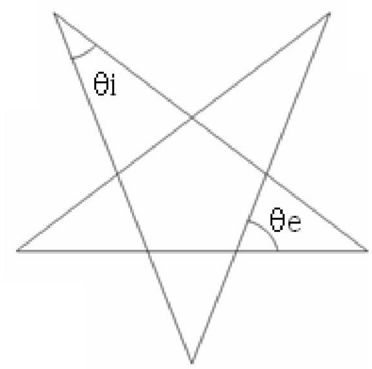
11. The sides of a regular polygon of n sides, n > 4, are extended to form a star. The number of degrees at each point of the star is :

(a) $\frac{360}{n}$ (b) $\frac{(n-4)180}{n}$

(c) $\frac{(n-2)180}{n}$ (d) $180-\frac{90}{n}$

Sol:

$Q_e = \frac{360}{n} + \frac{360}{n} + x = 180\,\theta i\,\theta e$



$x = 180 - \frac{2.360}{n} = \frac{(n-4)180}{n}$

12. Two equal parallel chords are drawn 8 inches apart in a circle of radius 8 inches. The area of that part of the circle that lies between the chords is:

(a) $21\frac{1}{3}\pi - 32\sqrt{3}$ (b) $32\sqrt{3} + 21\frac{1}{3}\pi$

(c) $32\sqrt{3} + 42\frac{2}{3}\pi$ (d) $16\sqrt{3} + 42\frac{2}{3}\pi$

Sol: b

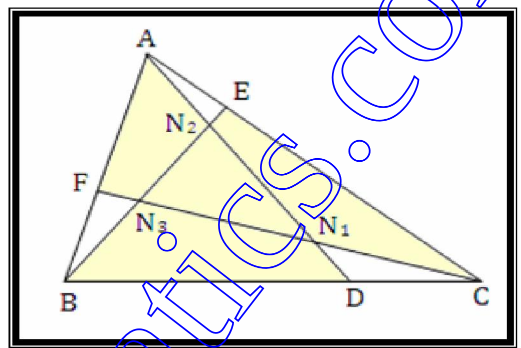
By symmetry, the required area is 4(T + S).

$T = \frac{1}{2}4.4\sqrt{3} = 8\sqrt{3},$

$S = \frac{30}{360}.\pi 8^2 = 5\frac{1}{3}\pi;$

$\therefore A = 32\sqrt{3} + 21\frac{1}{3}\pi.$

13. In the figure, $\overline{CD}, \overline{AE}$ and $\overline{BF}$ are one-third of their respective sides, It follows that $\overline{AN_2} : \overline{N_2N_1} : \overline{N_1D} = 3:3:1$, and similarly for lines BE and CF. Then the area of triangle $N_1N_2N_3$ is :



(a) $\frac{1}{10}\Delta ABC$ (b) $\frac{1}{9}\Delta ABC$

(c) $\frac{1}{7}\Delta ABC$ (d) $\frac{1}{6}\Delta ABC$

Sol:

By subtracting from ΔABC the sum of Δ CBF, ΔBAE and ΔACD and restoring ΔCDN_1 + ΔBFN_3 + Δ AEN_2, we have $\Delta N_1N_2N_3$

Δ CBF = ΔBAE = ΔACD = $\frac{1}{3}\Delta ABC$.

From the assertion made in the statement of the problem, it allows that

$\Delta CDN_1 = \Delta\ BFN_3 = \Delta\ AEN_2 = \frac{1}{7}.\frac{1}{3}\Delta ABC = \frac{1}{21}\Delta ABC.$

$\therefore\ \Delta N_1N_2\ N_3 = \Delta\ ABC - 3.\ \frac{1}{3}\Delta ABC + 3.\frac{1}{21}\Delta ABC = \frac{1}{7}\Delta ABC.$

14. A circular piece of metal of maximum size is cut out of a square piece and then a square piece of maximum size is cut out of the circular piece. The total amount of metal wasted is :

(a) $\frac{1}{4}$ the area of the original square (b) $\frac{1}{2}$ the area of the original square

(c) $\frac{1}{2}$ the area of the circular piece (d) $\frac{1}{4}$ the area of the circular piece

Sol: b

Let r be radius of one circle

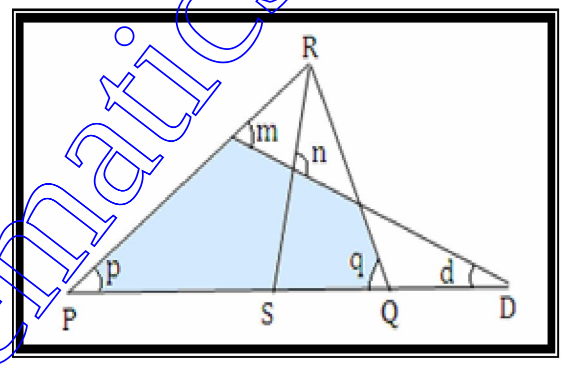
AO = OB = r

$AB = \sqrt{(AO)^2 + (OB)^2} = \sqrt{r^2 + r^2} = \sqrt{2r^2} = \sqrt{2}\,r$

radius of innermost circle = $\frac{1}{2}AB = \frac{1}{2}(\sqrt{2}\,r) = \frac{r}{\sqrt{2}}$

Area = $\pi\left(\frac{r}{\sqrt{2}}\right)^2 = \frac{\pi r^2}{2}$

15. Given triangle PQR with RS bisecting angle R, PQ extended to D and angle 'n' a right angle, then :



(a) $\angle m = \frac{1}{2}(\angle p - \angle q)$

(b) $\angle m = \frac{1}{2}(\angle p + \angle q)$

(c) $\angle d = \frac{1}{2}(\angle q + \angle p)$

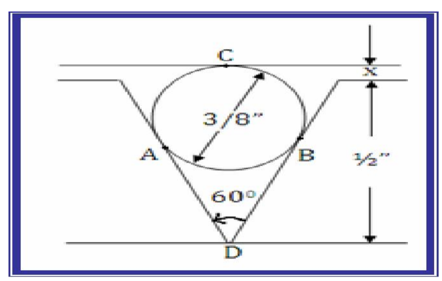
(d) $\angle d = \frac{1}{2}\angle m$

Sol:

$\measuredangle m = \measuredangle p + \measuredangle d,\ \measuredangle d = \measuredangle q - \measuredangle m$ (there are two vertical angles each $\measuredangle m$).

$\therefore\ \measuredangle m = \measuredangle p + \measuredangle q - \measuredangle m;\quad \therefore\ \measuredangle m = \frac{1}{2}(\measuredangle p + \measuredangle q)$.

16. In the diagram if points A, B, C are points of tangency, then x equals :



(a) $\frac{3''}{16}$ (b) $\frac{1''}{8}$

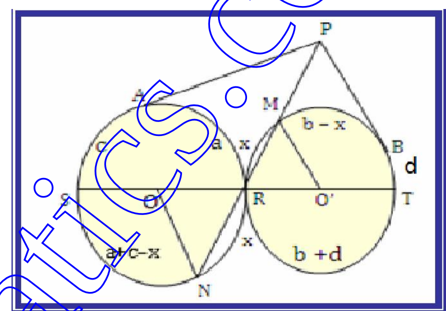
(c) $\frac{1''}{16}$ (d) $\frac{3''}{32}$

Sol:

The distance from the centre of the circle to the intersection point of the tangents is 3/8

$\therefore \quad \overline{CD} = \frac{3}{8} + \frac{3}{16} = \frac{9}{16} = x + \frac{1}{2}; \qquad \therefore x = \frac{1}{16}$ (cm).

17. In the figure PA is tangent to semicircle SAR; PB is tangent to semicircle RBT; SRT is a straight line; the arcs are indicated in the figure. Angle APB is measured by :



(a) $\frac{1}{2}$ (a – b) (b) (a + b)

(c) (c – a) (d) a – b

Sol: b

First, draw the line connecting P and R and denote its other intersections with the circles by M and N; see accompanying figure. The arcs MR and NR contain the same number of degrees; so we may denote each arc by x. To verify this, note that we have two isosceles triangles with a base angle of one equal to a base angle of the other.

$\therefore \ \measuredangle NOR = \measuredangle MO'R$

$$\measuredangle APR = \frac{1}{2}\{(c + a + c - x) - a\} = \frac{1}{2}\{2c - x\}$$

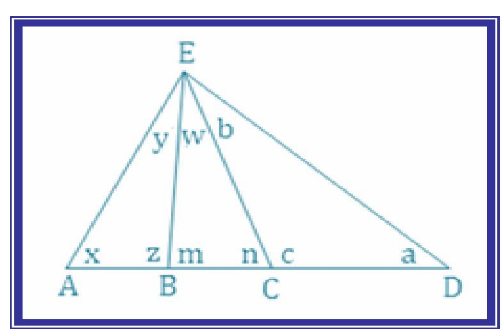
$$\measuredangle BPR = \frac{1}{2}\{b + d + d - (b - x)\} = \frac{1}{2}\{2d - x\}$$

and the sum of angles APR and BPR is $\measuredangle BPA = c + d$.

The desired angle is $360^o - \measuredangle BPA = 360^o - (c + d)$

$= (180^o - c) + (180^o - d) = a + b$

18. In a general triangle ADE (as shown) lines EB and EC are drawn. Which of the following angle relation is true ?



(a) x + z = a + b (b) y + z = a + b

(c) m + x = w + n (d) x + y + n = a + b +m

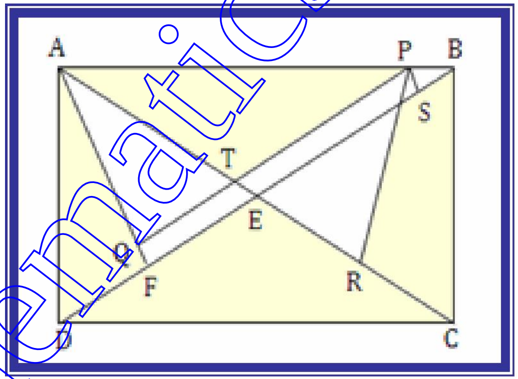
Sol: d

From triangle AEC, x + n +y + w = 180°

From triangle BED, m + a + w + b = 180°.

$\therefore$ x + y + n = a + b + m

19. ABCD is a rectangle (see the diagram) with P any point on AB. PS $\perp$ BD, PR $\perp$ AC, AF $\perp$ BD and PQ $\perp$ AF. Then PR + PS is equal to :



(a) $\overline{PQ}$ (b) $\overline{AE}$

(c) $\overline{PT}+\overline{AT}$ (d) $\overline{AF}$

Sol:

$\Delta\,PTR \sim \Delta\,ATQ;\ \overline{PR}/\overline{AQ} = \overline{PT}/\overline{AT}$

$\overline{PT}=\overline{AT}$ ($\measuredangle PAT = \measuredangle PBS = \measuredangle APT$); $\overline{PR}=\overline{AQ}, \overline{PS}=\overline{QF}$;

$\overline{PR} + \overline{\mathbf{PS}} = \overline{AQ} + \overline{QF} = \overline{AF}$; or

$\measuredangle SBP = \measuredangle TPA = \measuredangle TAP$, A, Q, R, P are concylic,

arc PR = arc AQ, $\overline{PR} = \overline{AQ}, \overline{PS} = \overline{QF}; \overline{PR} + \overline{RS} = \overline{AF}$.

20. The length of a triangle is of length b, and the altitude is of length h, A rectangle of height x is inscribed in the triangle with the base of the rectangle in the base of the triangle. The area of the rectangle is:

(a) $\frac{bx}{h}(h-x)$ (b) $\frac{hx}{b}(b-x)$

(c) $\frac{bx}{h}(h-2x)$ (d) x (b - x)

Sol:

Designate the base of the rectangle by y. Then,

because of similar triangles, $\frac{h-x}{y} = \frac{h}{b}$, $y = \frac{b}{h}(h-x)$

$\therefore$ area = xy = $\frac{bx}{h}$ (h - x)

21. Compute $1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots - 1998^2 + 1999^2$.

(a) 1,999,000 (b) 1,888,000

(c) 1,999,999 (d) 2,999,999

Sol: a

$s = 1999^2 - 1998^2 + 1997^2 - 1996^2 + \ldots\ldots + 3^2 - 2^2 + 1^2$

= (1999 + 1998) + (1999 - 1998) + (1997 + 1996) + (1997 - 1996) + ... + (3 + 2) + (3 - 2) + 1

= 1999 + 1998 + 1997 + 1996 + ... + 3 + 2 + 1

$= \frac{1999 \,.\, 2000}{2} = 1,999,000$

22. What remainder are obtained when the number consisting of 1001 sevens is divided by the number 1001 ?

(a) 777 (b) 707

(c) 700 (d) 770

Sol:

The number 777, 777 is exactly divisible by 1001, yielding the quotient 777. Hence the number

$\underbrace{777\ldots\ldots\ 700000}$, yields, upon division by 1001, a quotient of

$\underbrace{777000\ 777\ 000\ldots\ldots\ 7777\ 000\ 00}$

the grouping 777, 000 repeated 1666 times

Moreover, the number 77, 777 yields a quotient of 777 and a remainder of 7000 upon division by 1001; and so the quotient obtained by dividing A by 1001 has the form

$\underbrace{777000\ 777\ 000.......\ 7777\ 000\ 77}$

the grouping 777, 000 repeated 166 times and there is a remainder of 700.

23. Compute the unique positive integer n such that

$$2.2^2 + 3.2^3 + 4.2^4 + 5.2^5 + ... + n.2^n = 2^{(n+10)}$$

(a) 403 (b) 513

(c) 413 (d) 503

Sol: b

$(1.2^1) + 2 . 2^2 + 3.2^3 + ... + n . 2^n$

$= 2^{(n+10)} + (2)$. The left side can be summed as:

$2^1 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$

$2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2^2$

$2^3 + ... + 2^n = 2^{n+1} - 2^3$

.... = ...

$+2^n = \dfrac{2^{n+1} - 2^n}{n(2^{n+1}) - (2^{n+1} - 2)}$

$= 2^{n+1}(n - 1) + 2.$

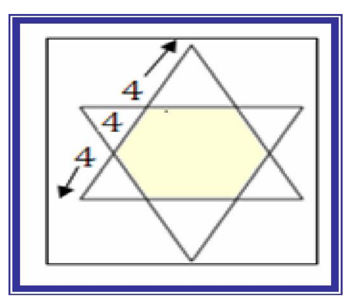
since this equals

$2^{n+10} + 2,\ n - 1 = \dfrac{2^{n+10}}{2^{n+1}}$, so $n = 2^9 + 1 = 513$

24. Two equilateral triangle measures 12 cm on each side. They are positioned to form a regular six-pointed star. What is the area of the overlapping figure?

(a) $48\sqrt{3}\ cm^2$ (b) $24\sqrt{3}\ cm^2$

(c) $36\sqrt{3}\ cm^2$ (d) $12\sqrt{3}\ cm^2$



Sol:

$$\text{Required Area } = \frac{\sqrt{3}}{4}(12)^2 - 3\left(\frac{\sqrt{3}}{4}(4)^2\right)$$

$$\frac{\sqrt{3}}{4}\times 144 - \frac{3\sqrt{3}}{4}(16)$$

$$\sqrt{3}(36-12)=24\sqrt{3}\,c$$

25. A digital clock displays the correct time on 1 January 2012. If the clock loses 15 minutes per day, what will be the next date when the clock displays the correct time?

(a) 7^{th} april (b) 17^{th} feb

(c) 6^{th} april (d) 18^{th} feb

Sol:

The clock loses 1 hour in 4 days so it loses 12 hours in 48 days. From 1 January to 1 February is 31 days. Add 17 days to get to 18 February, assuming that only a 12-hour clock was used.

But the problem describes a digital clock specifically. Since virtually all digital clocks show 24-hours or A.M through P.M., this solution is the only one possible. If we assume that a 24 hour clock was used, then solution is 96 days. That is, the 97th day of the year = 31 + 28 = 31 = 90, 90 + 7, which is 7 April on 6 April in a leap year. Consider a clock that also shows the month and the year

26. Sachin Verma said: "The day before yesterday I was 10, but I will turn ……(?)…….yrs in the next year" {maximum possible answer is}

(a) 12 (b) 13

(c) 11 (d)14

Sol:

We need to have as much time as possible between the uttering of this sentence and Peter's next birthday. We can manage this if he made this statement on

January 1, and he was born on December 31. He will turn 13 at the end of the next calendar year

27. The remainder when 7^{84} is divided by 342 is

(a) 0 (b) 1

(c) 49 (d) 341

Sol:b

$7^3 = 343$,when divided by 342, leaves a remainder of 1

7^4 = 2401, when divided by 342, leaves a remainder of 7.

7^5 = 16807, when divided by 342, leaves a remainder of 49.

7^6 = 117649, when divided by 342, leaves a remainder of 1.

And so on.

$\therefore$ 7^{84}, when divided by 342, will leave a remainder of 1

28. $\sqrt{6+2\sqrt{2}+2\sqrt{3}+2\sqrt{6}}-\frac{1}{\sqrt{5-2\sqrt{6}}}$ is Equal to

(a) 1 (b) $\sqrt{2}$

(c) $6\sqrt{2}$ (d) $2\sqrt{6}$

Sol:1

$$\sqrt{(1)^2+(\sqrt{2})^2+(\sqrt{3})^2+2(1)(\sqrt{2})+2(1)(\sqrt{3})+2(\sqrt{2})(\sqrt{3})}$$

$$-\frac{1}{\sqrt{(\sqrt{3})^2+(\sqrt{2})^2-2\sqrt{2}.\sqrt{3}}}$$

$$(1+\sqrt{2}+\sqrt{3})^2-\frac{1}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$\frac{1+\sqrt{2}+\sqrt{3}}{1}-\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$\frac{1+\sqrt{2}+\sqrt{3}}{1}-\frac{\sqrt{3}+\sqrt{2}}{1}$$

$$\frac{1+\sqrt{2}+\sqrt{3}}{1}-\sqrt{2}-\sqrt{3}=1$$

29. If f(x) + f(1 – x) = 1. Then

$$f\left(\frac{1}{1997}\right)+f\left(\frac{2}{1997}\right)+\ldots\ldots\ldots\ldots\ldots+f\left(\frac{1996}{1997}\right)=?$$

(a) 999 (b) 998

(c) 919 (d) 918

Sol:

$$=f\left(\frac{1}{1997}\right)+f\left(\frac{1996}{1997}\right)+f\left(\frac{2}{1997}\right)+f\left(\frac{1995}{1997}\right)+-----$$

$$=f\left(\frac{1}{1997}\right)+f\left(1-\frac{1}{1997}\right)+f\left(\frac{2}{1997}\right)+f\left(1-\frac{2}{1997}\right)$$

= 1 + 1 + 1 --------- + 1 (998 times) (f(x) + f(1 – x) = 1 by question)

$= \frac{1996}{2} = 998$

= 998

30. $32^{32^{32}}$ when divided by 7 leaves remainder

(a) 2 (b) 3

(c) 4 (d) 5

Sol: (c)

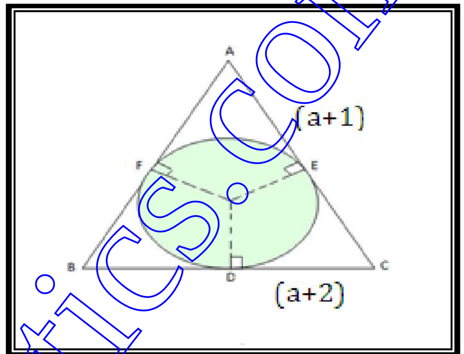
Award winning Question.

Give the solution of the question and get your reward from pioneermathematics.com.

31. In triangle ABC, the incircle touches the sides, BC, CA and AB at D, E F respectively. If radius of incircle is 4 units and BD, CE, AF be consecutive integers, find the sides of triangle ABC.

(a) 9, 10, 11 (b) 13, 14, 15

(c) 14, 15, 16 (d) None of these



Sol:

Area of ΔABC By Hero's form

$$S = \frac{a+a+1+a+2}{2}$$

$$= \frac{3a+3}{2} = \frac{3}{2}(a+1)$$

$$D = \sqrt{\frac{3}{2}(a+1)\left[\frac{3}{2}(a+1) - a\right]\left[\frac{3}{2}(a+1)-(a+1)\right]\left[\frac{3}{2}(a+1)-(a+2)\right]}$$

$$= \sqrt{\frac{3}{2}(a+1)\left(\frac{3a+3-2a}{2}\right)\left(\frac{3a+3-2a-2}{2}\right)}\left(\frac{3a+3-2a-4}{2}\right)$$

$$\sqrt{\frac{3}{2}(a+1)\left(\frac{a+3}{2}\right]\left(\frac{a+1}{2}\right)\left(\frac{a-1}{2}\right)}$$

$$\frac{1}{4}\sqrt{(a+1)(a+3)(a+1)(a-1)} \quad ....(1)$$

Now, Area of ΔABC

Now, $= \frac{1}{2}[4a] + \frac{1}{2}(4(a+2)) + \frac{1}{2}(4(a+1))$

$$= \frac{1}{2}(4)(a+a+1+a+2).$$

= 2 (3a + 3)

= 6 (a + 1). ....(2)

from (1) and (2)

$$\frac{1}{6}(a+1)^2(a+3)(a-1) = 36(a+1)^2$$

(a + 3) (a – 1) = 12 × 16

a = 13

32. The interior angle of a regular polygon exceeds the exterior angel by 132^0. The number of sides in the polygon is ……….

(a) 7 (b) 8

(c) 12 (d) 15

Sol:

$$\theta_i = \frac{(n-2)}{n}\times 180 \quad \theta_e = \frac{360}{n} \quad \theta_i - \theta_e = 132^0$$

$$\frac{(n-2)180}{m} - \frac{360}{n} = 132^0.$$

$$\frac{180n-360}{n} - \frac{360^0}{n} = 132^0$$

$$\frac{180n-360^0-360^0}{n} = 132^0$$

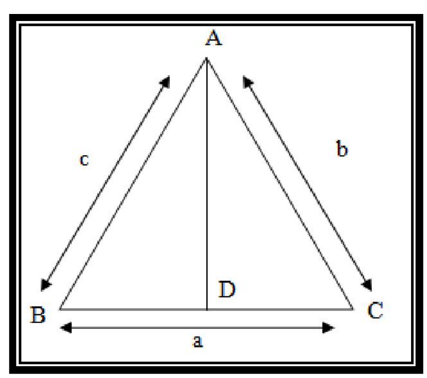
$$\frac{180n-720}{n} = 132^0$$

$$180n-720-132n = 0$$

$$48n-720 = 0$$

$$n = \frac{720}{48} = 15$$

33. Given the sides of a triangle (a, b and c). then the median m_a drawn to the side 'a ' can be GIVEN by the formula



(a) $m_a = \frac{1}{2}\sqrt{2b^2-c^2+2a^2}$ (b) $m_a = \frac{1}{2}\sqrt{2c^2+2a^2-b^2}$

(c) $m_a = \frac{1}{2}\sqrt{b^2+c^2+a^2}$ (d) $m_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

Sol:

. In ΔABC, the mid-points of the sides BC, CA and AB are D, E and F respectively. The lines, AD, BE and CF are called medians of the triangle ABC, the points of concurrency of three medians is called centroid. Generally it is represented by G.

By analytical geometry :

$$AG = \frac{2}{3} AD$$

$$BG = \frac{2}{3} BE$$

and $$CG = \frac{2}{3} CF$$

Length of medians and the angles that the median makes with sides

In above figure,

$AD^2 = AC^2 + CD^2 - 2AC.CD. \cos C$

$$\therefore \; AD^2 = b^2 + \frac{a^2}{4} - ab \cos C$$

$$\therefore \; AD^2 = b^2 + \frac{a^2}{4} - ab.\left(\frac{b^2 + a^2 - c^2}{2ab}\right)$$

$$AD^2 = \frac{2b^2 + 2c^2 - a^2}{4}$$

$$\Rightarrow \; AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

or $$AD = \frac{1}{2}\sqrt{b^2 + c^2 + 2bc \cos A}$$

34. Pedal triangle is a triangle formed by joining the foot of the altitudes in a triangle ,then the orthocentre of a triangle is the ……………….of the pedal triangle.

(a) circumcentre (b) Incentre

(c) Centroid (d) orthocenter

Sol: (b)

Incentric

35. Find the highest power of 3 contained in 1000!

(a) 499 (b) 498

(c) 496 (d) 497

Sol: (b)

P = 3, n = 1000

$$\left[\frac{n}{p^5}\right] = \left[\frac{12}{3}\right] = [4] = 4$$

$$\left[\frac{n}{p^6}\right] = \left[\frac{4}{3}\right] = \left[1\frac{1}{3}\right] = 1$$

$$\left[\frac{n}{p^7}\right] = \left[\frac{1}{3}\right] = 0$$

$$\left[\frac{n}{p}\right] = \left[\frac{1000}{3}\right] = \left[333\frac{1}{3}\right] = 333$$

$$\left[\frac{n}{p^2}\right] = \left[\frac{333}{3}\right] = [111] = 111$$

$$\left[\frac{n}{p^3}\right] = \left[\frac{111}{3}\right] = [37] = 37$$

$$\left[\frac{n}{p^4}\right] = \left[\frac{37}{3}\right] = \left[12\frac{1}{3}\right] = 12$$

$\therefore$ Highest power of 3 contained in 1000!

$$= \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \left[\frac{n}{p^4}\right] + \left[\frac{n}{p^5}\right] + \left[\frac{n}{p^6}\right] + \left[\frac{n}{p^7}\right]$$

= 333 + 111 + 37 + 12 + 4 + 1 + 0 = 498

36. Find the remainder when $2^{100} + 3^{100} + 4^{100} + 5^{100}$ is divided by 7

(a) 2 (b) 5

(c) 6 (d) 3

Sol: (b)

$2^{100} = 2 \bmod 7$

$3^{100} = 4 \bmod 7, \bmod m$

$4^{100} = 4 \bmod 7,$

$5^{100} = 2 \bmod 7,$

$\therefore 2^{100} + 3^{100} + 4^{100} + 5^{100} = 12 \bmod 7$ $\left\{\begin{array}{l} \therefore a \equiv b \bmod m \\ \quad c \equiv d \bmod m \\ a + c \equiv b + d \bmod m \end{array}\right\}$

But $12 \equiv 5 \bmod 7$

$\therefore 2^{100} + 3^{100} + 4^{100} + 5^{100} \equiv 5 \bmod 7$

$\therefore$ Remainder is 5

37. If α, β, γ be the roots of $x^3 + 2x^2 - 3x - 1 = 0$.Find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3}$.

(a) -40 (b) -41

(c) -42 (d) None of these

Sol: (c)

Roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$

are α, β, γ.

Let us first form an equation whose roots are $\alpha^3, \beta^3, \gamma^3$.

If y is root of the transformed equation, then

$$y = x^3$$

To eliminate x between Eqs. (i) and (ii). Eq. (i) can be written as

$x^3 - 1 = -(2x^2 - 3x)$

On cubing both sides of above equations

$\Rightarrow \quad x^9 - 3x^6 + 3x^3 - 1 = -[8x^6 - 27x^3 - 18x^3(2x^2 - 3x)]$

$\Rightarrow \quad x^9 - 3x^6 + 3x^3 - 1 = -[8x^6 - 27x^3 - 18x^3(1 - x^3)]$

$\Rightarrow \quad x^9 - 3x^6 + 3x^3 - 1 = -8x^6 + 27x^3 + 18x^3 - 18x^6$

$\Rightarrow \quad x^9 + 23x^6 - 42x^3 - 1 = 0$

Putting $x^3 = y$ in this equation, we get

$$y^3 + 23y^2 - 42y - 1 = 0$$

it roots are

$\alpha^3, \beta^3, \gamma^3$.

Changing y to $\frac{1}{y}$, Eq. (iii) becomes

$$\frac{1}{y^3}+\frac{23}{y^2}-\frac{42}{y}-1=0$$

$$y^3 + 42y^2 - 23y - 1 = 0$$

Its roots are $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}$.

$\therefore$ $\frac{1}{\alpha^3}, \frac{1}{\beta^3}, \frac{1}{\gamma^3}$. = sum of roots of Eq. (iv) = - 42

38. If we Solve the system

$$\frac{b(x+y)}{x+y+cxy}+\frac{c(z+x)}{x+z+bxz}=a$$

$$\frac{c(y+z)}{y+z+ayz}+\frac{a(x+y)}{x+y+cxy}=b$$

$$\frac{a(x+z)}{x+z+bxz}+\frac{b(y+z)}{y+z+ayz}=c.$$

then the value of x=

(a) $=\frac{a}{a+b+c}$.

(b) $=-\frac{a}{a+b+c}$.

(c) $=\frac{a}{a-b-c}$.

(d) None of these

Sol:

Put $\frac{x+y}{x+y+cxy}=\gamma$; $\frac{y+z}{y+z+ayz}=\alpha$

and $\frac{x+z}{x+z+bxz}=\beta$

Then, the system takes the form

$b\gamma + c\beta = a$; $c\alpha+\alpha\gamma=b$; $\alpha\beta+b\alpha=c$

or $\frac{\gamma}{c}+\frac{\beta}{b}=\frac{a}{bc};\frac{\alpha}{a}+\frac{\gamma}{c}=\frac{b}{ac};$

$$\frac{\beta}{b}+\frac{\alpha}{a}=\frac{c}{ab}$$

$$\therefore\ \frac{\alpha}{b}+\frac{\beta}{b}+\frac{\gamma}{c}=\frac{1}{2}\ abc\frac{a^2+b^2+c^2}{abc}$$

and consequently

$$\alpha=\frac{b^2+c^2-a^2}{2bc};\beta=\frac{a^2+c^2-b^2}{2ac};$$

$$\gamma=\frac{a^2+b^2-c^2}{2ab}$$

Further $\frac{x+y+cxy}{x+y}=\frac{1}{\gamma}$

or $\frac{cx}{x+y}=\frac{1}{\gamma}-1$

or $\frac{x+y}{cxy}=\frac{\gamma}{1-\gamma}$

Finally $\frac{1}{x}+\frac{1}{y}=\frac{c\gamma}{1-\gamma}$

Analogously, we get

$$\frac{1}{x}+\frac{1}{z}=\frac{b\beta}{1-\beta};\frac{1}{y}+\frac{1}{z}=\frac{a\alpha}{1-\alpha}$$

Where from we find x, y, z

39. Which of the following TRUE

(a) $(31)^{12}>(17)^{17}$

(b) $(30)^{100}>(2)^{567}$

(c) $7^{92}>8^{91}$

(d) $(150)^{300}>(20000)^{100}\times(100)^{100}$

Sol: (d)

(1) Now, 31 < 32

Raising the power to 12

$(31)^{12} < (32)^{12}$

$\Rightarrow \quad (31)^{12} < (2^5)^{12} = 2^{60} \quad ...(i)$

Now, $2^{60} < 2^{68} = 2^{4\times17} \quad ...(ii)$

$\Rightarrow \ (2^4)^{17} < (16)^{17}$

$\Rightarrow \quad (16)^{17} < (17)^{17} \quad ...(iii)$

From (i), (ii) and (iii),we get

$(31)^{12} < 2^{60} < 2^{68} < (17)^{17}$

$\Rightarrow \quad (31)^{12} < (17)^{17}$

$\therefore \ (17)^{17} > (31)^{12}$

(ii) $(30)^{100} < (32)^{100}$

So, $(2^5)^{100} = (2^{10})^{50} = (1024)^{50}$

Now, $(1024)^{50} < (1024)^{54} = ((1024)^2)^{27} = (2^{20})^{27}$

Now, $2^{20} < 2^{21}$

$(2^{20})^{27} < (2^{21})^{27} = 2^{567}$

$\therefore \quad (30)^{100} < 2^{567}$

$\therefore \ 2^{567} > (30)^{100}$

(iii) If a, b are +ve and n is a natural number, then

$(a + b)^n = a^n + na^{n-1}b + ...$

(term involving powers of a and b).

Also, $(a + b)^n \geq a^n + na^{n-1}b$ (equality for n = 1)

now, $(8)^{91} = (7 + 1)^{91} > 7^{91} + 91.7^{90}$

$= 7^{90}(7 + 91) = 7^{90}(98)$

Now, $7^{90}(98) > 7^{90}.49 = 7^{90}.7^2 = 7^2$

Hence, $(8)^{91} > 7^{92}$

(iv) We know $(150)^3 = 150 \times 150 \times 150$

$= 30 \times 30 \times 30 \times 125 = 27000 \times 125$

Now, $27000 \times 125 > 20000 \times 100$

Hence, $(150)^3 > 20000 \times 100$

Raising the power to 100

$(150)^{300} > (20000)^{100} \times (100)^{100}$

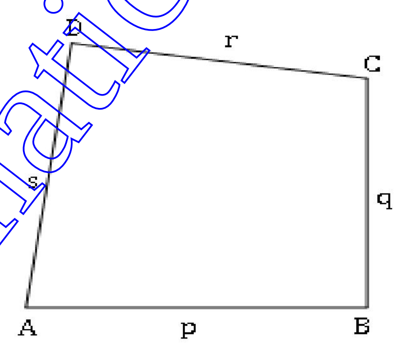
40. If P, Q, R, S are the sides of a quadrilateral. Find the minimum value of

$$\frac{p^2+q^2+r^2}{s^2}$$

(a) 1/2 (b) 1/3

(c) 2/3 (d) 3/2

Sol:



We have

AB = p

BC = q

CD = r

AD = s

We know that

$(p - q)^2 + (q - r)^2 + (r - p)^2 \geq 0$

$\Rightarrow \quad 2(p^2 + q^2 + r^2) \geq 2(pq + qr + rp)$

$\Rightarrow \quad 3(p^2 + q^2 + r^2) \geq (p^2 + q^2 + r^2) + 2(pq + qr + rp)$

[on adding $p^2 + q^2 + r^2$ to both sides]

$\Rightarrow \quad 3(p^2 + q^2 + r^2) \geq (p + q + r)^2$

[∵ sum of any three sides of a quadrilateral is greater than fourth one]

$\Rightarrow \quad 3(p^2 + q^2 + r^2) \geq (p + q + r)^2 > s^2$

$$\Rightarrow \quad \frac{p^2 + q^2 + r^2}{s^2} > \frac{1}{3}$$

∴ Minimum value of $\frac{p^2 + q^2 + r^2}{s^2}$ is $\frac{1}{3}$

41. A student on vacation for D days observed that

(i) it rained 7 times morning or afternoon

(ii) when it rained in the afternoon it was clear in the morning

(iii) there were 5 clear afternoons, and

(iv) there were 6 clear mornings.

Find D.

(a)9 (b) 7

(c)10 (d)5

Sol:

Let the set of days it rained in the morning be M2.

Let A_r be the set of days it rained in afternoon.

M' be the set of days, when there were clear morning.

A'_r be the set of days when there were clear afternoon.

By condition (ii), we get $M_r \cap A_r = \phi$

By (iv), we get $M'_r = 6$

By (iii), we get $A'_r = 5$

By (i), we get $M_r \cup A_r = 5$

M_r and A_r are disjoint sets $n(M_r) = d - 6$

$n(A_r) = d - 5$

Applying the principal of inclusion-exclusion

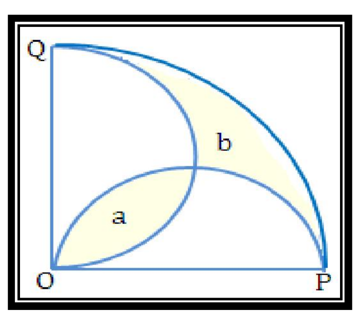
$$n(M_r \cup A_r) = n(M_r) + n(A_r) - n(M_r \cap A_r)$$

$\Rightarrow \quad 7 = (d - 6) + (d - 5) - 0$

$\Rightarrow \quad 2d = 18$

$\Rightarrow \quad d = 9$

42. OPQ is a quadrant of a circle and semicircles are drawn on OP and OQ. Then



(a)a>b (b) a<b

(c)a=b (d)can't be determined

Sol: (c)

Area of quadrant = areas of two semi circles + b – a.

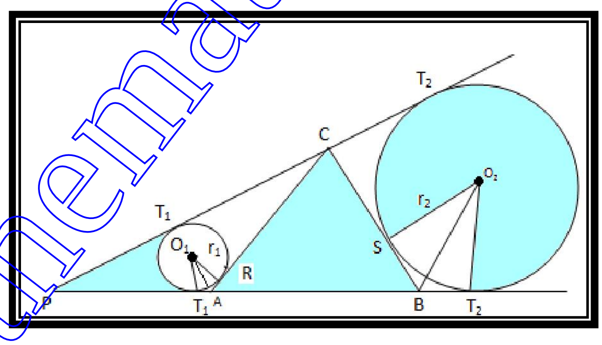
i.e., $\frac{1}{4}\pi r^2 = \frac{1}{2}\pi\left(\frac{r^2}{2}\right) + \frac{1}{2}\pi\left(\frac{r}{2}\right)^2 + b - a$

$\Rightarrow \quad \frac{1}{4}\pi r^2 = \frac{1}{4}\pi r^2 + b - a$

$\Rightarrow \quad b - a = 0$

$\Rightarrow \quad a = b$

43. Let Δ ABC be equilateral. On side AB produced, we choose a point P such that A lies between P and B. We now denote a as the length of sides of Δ ABC; r_1 as the radius of incircle of Δ PAC; and r_2 as the extradius of Δ PBC with respect to side BC. Determine the sum $r_1 + r_2$ as a function of 'a' alone.



(a) $\frac{a\sqrt{3}}{2}$ (b) $\frac{a\sqrt{2}}{3}$

(c) $\frac{a\sqrt{5}}{3}$ (d) $3\sqrt{3}a$

Sol: (a)

we see that $\angle T_1O_1R = 60^o$ since it is the supplement of $\angle T_1AR = 120^o$ (as an exterior angle for ΔABC). Hence, $\angle AO_1R = 30^o$. Similarly, we obtain $\angle BO_2S = 30^o$. Since, tangents drawn to a circle an external point are equal, we have

$T_1T_2 = T_1A + AB + BT_2$

$= RA + AB + SB$

$= r_1 \tan 30^o + a + r_2 \tan 30^o = \frac{r_1 + r_2}{\sqrt{3}} + a$

and $T_1'T_2' = T_1'C + CT_2'$

$= CR + CS = (a - RA) + (a - SB) = 2a - \frac{r_1 + r_2}{\sqrt{3}}$

Since, common external tangents to two circles are equal

$T_1T_2 = T_1^{'}T_2^{'}$

Hence, $\frac{r_1 + r_2}{\sqrt{3}} + a = 2a - \frac{r_1 + r_2}{\sqrt{3}}$,

Whence we find that

$r_1 + r_2 = \frac{a\sqrt{3}}{2}$

44. The sum of n term of the series

$$\frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{9}} + \ldots\ldots \text{ is}$$

(a) $\sqrt{2n+3}$

(b) $\frac{\sqrt{2n+3}}{2}$

(c) $\sqrt{2n+3} - \sqrt{3}$

(d) $\frac{\sqrt{2n+3} - \sqrt{3}}{2}$

Sol:

$$\frac{1}{\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1}{\sqrt{5}+\sqrt{7}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{7}+\sqrt{9}} \times \frac{\sqrt{9}-\sqrt{7}}{\sqrt{9}-\sqrt{7}} \ldots\ldots\ldots\ldots$$

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{5-3} + \frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{9}-\sqrt{7}}{9-7} \ldots\ldots\ldots \frac{1}{\sqrt{2n+3}-\sqrt{2n+1}} \times \frac{\sqrt{2n+3}+\sqrt{2n+1}}{\sqrt{2n+3}+\sqrt{2n+1}}$$

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \frac{\sqrt{9}+\sqrt{7}}{2} + \text{-------} \frac{\sqrt{2n+3}+\sqrt{2n+1}}{2n+3-2n-1} = \frac{\sqrt{2n+3}+\sqrt{2n+1}}{2}$$

$$\Rightarrow \frac{1}{2}\left[\sqrt{5} - \sqrt{3} + \sqrt{7} - \sqrt{5} \text{ ------} \sqrt{2n+3} - \sqrt{2n+1}\right]$$

$$\Rightarrow \frac{1}{2}\left[\sqrt{2n+3} - \sqrt{3}\right]$$

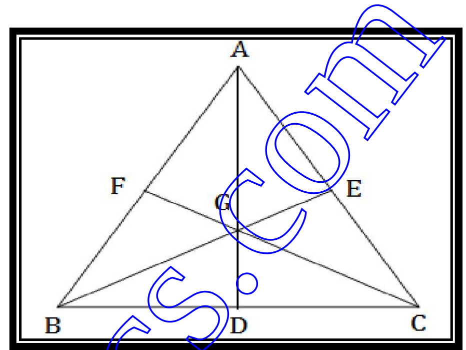
45. In ΔABC, the medians AD, BE and CF meet at G, then

(a) 4(AD + BE + CF) > 3(AB + BC + AC)

(b) 3(AD + BE + CF) > 2(AB + BC + AC)

(c) 3(AD + BE + CF) > 4(AB + BC + AC)

(d) 2(AD + BE + CF) >3 (AB + BC + AC)



Sol: (a)

4(AD + BE + CF) > 3(AB + BC + AC)

46. The point of concurrency of the perpendicular bisectors of a triangle is called

(a) Incentre (b) Orthocentre

(c) Circumcentre (d) Centroid

Sol: (c)

Circumcentre

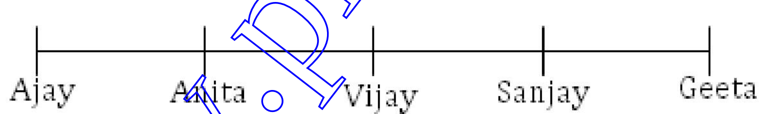
47. Some friends are sitting on a bench. Vijay is sitting next to Anita and Sanjay is next to Geeta .Geeta is not sitting with Ajay. Ajay is on the left end of the bench and Sanjay is in second position from right hand side. Vijay is on the right side of Anita and to the right side of Ajay, Vijay and Sanjay are sitting together.

Who is sitting in the centre?

(a) Ajay (b) Vijay

(c) Geeta (d) Sanjay

Sol: (b)



48. If x – k divides $x^3 - 6x^2 + 11x - 6 = 0$, then k can't be equal to

(a) 1 (b) 2

(c) 3 (d) 4

Sol: (d)

$\rightarrow$ x = k is zero of polynomial

Now put k = 1

$1^3 - 6(1)^2 + 11(1) - 6 = 0$

$1 - 6 + 11 - 6 = 0$

$0 = 0$

k = 2

$(2)^3 - 6(2)^2 + 11 \times 2 - 6 = 0$

$8 - 24 + 22 - 6 = 0$

$2 - 2 = 0$

k = 3

$(3)^3 - 6(3)^2 + 11 \times 3 - 6 = 0$

$27 - 54 + 33 - 6 = 0$

$21 - 21 = 0$

k = 4

$(4)^3 - 4(4)^2 + 11 \times 4 - 6 = 0$

$64 - 96 + 44 - 6 = 0$

$56 - 42 = 0$

k = 14

49. A convex polygon has 44 diagonals. The number of its sides is

(a) 10 (b) 11

(c) 12 (d) 13

Sol: (b)

$$^{n}C_2 - n = \frac{n(n-1)}{2} - n$$

No. of diagonal. = $\frac{n(n-1)-2n}{2}$

$44 = \frac{n^2 - n - 2n}{2}$

$88 = n^2 - 3n$

$88 = n(n - 3)$

$88 = 11 \times 8 = 11\ (11 - 3)$

$\therefore \quad n = 11$

50. Roman numeral for the greatest three digit number is

(a) IXIXIX (b) CMXCIX

(c) CMIXIX (d) CMIIC

Sol: (b)

CMXCIX

51. In the new budget, the price of a petrol rose by 10%, the percent by which one must reduce the consumption so that the expenditure does not increase is :

(a) $6\frac{1}{9}\%$ (b) $6\frac{1}{4}\%$

(c) $9\frac{1}{11}\%$ (d) 10%

Sol: (c)

Let price of petrol = Rs x

price hike = 10%

i.e. $\frac{10}{100} \times x = \frac{x}{10}$

New price = $x + \frac{x}{10} = \frac{11x}{10}$

earlier consumption = y litra

earlier investment = xy.

A.T.Q.,

Present investment = previous investment

$\left(\frac{11x}{10}\right)$ (present petrol consumption) = xy present petrol consumption = (xy) ×

$\frac{10}{11x} = \frac{10y}{11}$

Reduction in consumption = y - $\frac{10y}{11}$ = y/11 % age = $\frac{y/11 \times 100}{y}$

$= \frac{100}{11} = 9\frac{1}{11}\%$

52. Like dozen is 12 articles ,What is "score" equals to

(a) 20 (b) 30

(c) 24 (d) 36

Sol: (a)

20

53. Three traffic lights at three different road crossing change after 48 seconds, 72 seconds and 100 seconds respectively, If they all change simultaneously at 8 a. m., at what time will they again change simultaneously?

(a) 10 a.m. (b) 9 a.m.

(c) 11 a.m. (d) 10.30 a.m.

Sol: (b)

L.C.M of 48, 72, 100

is = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5

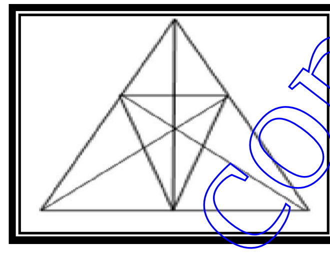
= 3600 sec

48 = 2 × 2 × 2 × 2 × 3 = 1 hour

72 = 2 × 2 × 2 × 3 × 3

100 = 2 × 2 × 5 × 5

54. Tell the number of triangles in the following figures



(a)40 (b) 45

(c) 47 (d) 50

Sol: (c)

47

55. A school bus travels from Delhi to Chandigarh. There are 4 children, 1 teacher and 1 driver in the bus. Each child has 4 backpacks with him. There are 4 dogs sitting in each backpack and every dog has 4 puppies. What is the total number of eyes in the bus.

(a) 256 (b) 128

(c) 657 (d) 652

Sol: (d)

No. of teacher =1

No. of driver= 1

eyes of teacher and driver= (1+1)X2=4

No. of children=4

eyes of children= 4 x 2=8

No. of dogs in each backpack= 4x4=16x4=64x2=128 eyes

eyes of puppies= 64x4=256x2=512 eyes

Total eyes= 4+8+128+512=652 eyes

56. Ravi is not wearing white and Ajay is not wearing blue. Ravi and sohan wear different colour. Sachin alone wear red. What is sohan colured, if all four them are wearing different colour.

(a) red (b) blue

(c) white (d) can't say

Sol: (d)

The fourth colour and some more information are required

57. Who is the father of Geometry ?

(a) Pythagoras (b) Thales

(c) Archimedes (d) Euclid.

Sol: (d)

Euclid.

58. Which of the following correctly shows 185367249 according to International place value chart ?

(a) 1, 853, 672, 49 (b) 18, 536, 724, 9

(c) 185, 367, 249 (d) None of these

Sol: (c)

185, 367, 249

59. (x% of y + y% of x) =

(a) x% of y (b) y% of x

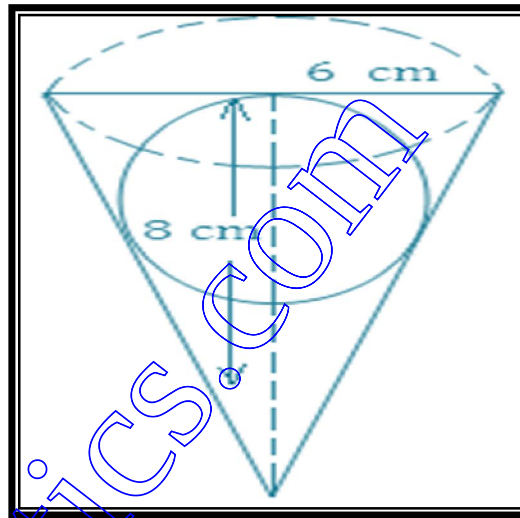
(c) 2% of xy (d) x% of xy

Sol:

$$\frac{x}{100} \times y + \frac{y}{100} \times x$$

$$= \frac{2xy}{100} = \frac{2}{100} \times xy$$

60. A conical vessel of radius 6 cm and height 8 cm is completely filled with water.A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed. What fraction of the water overflows?



(a) $\frac{2}{5}$ (b) $\frac{3}{8}$

(c) $\frac{3}{5}$ (d) $\frac{3}{4}$

Sol: (b)

A vertical section of the conical vessel and the sphere when immersed are shown in the figure.

From right angled Δ AMB,

$AB^2 = AM^2 + MB^2 = 82 + 62$

$= 64 + 36 = 0$

$\Rightarrow$ AB = 10 cm.

CB is tangent to the circle at M and AB is tangent to it at P.

PB = MB = 6

($\therefore$ lengths of tangents from an external point to a circle are equal in length)

$\therefore$ AP = AB - PB = (10 – 6) cm = 4 cm.

Let r cm be the radius of the circle, then OP = OM = r

$\therefore$ AO = AM – OM = (8 – r) cm.

From right angled Δ OAP,

$OA^2 = AP^2 + OP^2$

$\Rightarrow (8 - r)^2 = 42 + r^2$

$\Rightarrow 64 - 16r + r^2 = 16 + r^2$

$\Rightarrow$ 48 = 16r $\Rightarrow$ r = 3.

$\therefore$ Radius of circle i.e. of the sphere = 3 cm.

$\therefore$ Volume of sphere = $\frac{4}{3}\pi \times 3^3 \text{ cm}^3 = 36\pi \text{ cm}^3$.

The volume of water which overflows = volume of the sphere

= 36π cm^3.

Volume of water in the cone before immersing the sphere

= volume of the cone = $\frac{1}{3}\pi \times 6^2 \times 8$ cm^3

= 96π cm^3.

$\therefore$ The fraction of water which overflows = $\frac{\text{Volume of water overflows}}{\text{Total volume of water}} = \frac{36\pi}{96\pi} = \frac{3}{8}$.