+2 (FLT - 1)

Time: 3hrs.   **Marks: 100**

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) Question numbers 1 to 10 are very short answer questions and carry 1 mark each,

(iii) Question numbers 11 to 22 are short answer questions and carry 4 marks each,

(iv) Question numbers 23 to 29 are long answer type questions and carry 6 marks each.

Section - A

1. Give an example to show that the relation R in the set of natural numbers, defined by R = {(x, y), x, y, $\in$ N, x $\leq$ y^2} is not transitive.

2. Using principal values, write the value of $2\cos^{-1}\dfrac{1}{2}+3\sin^{-1}\dfrac{1}{2}$.

3. For what value of k, the matrix $\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible?

4. Evaluate: $\int \dfrac{\sec x \tan x}{\sqrt{4-\sec^2 x}}dx$.

5. Write a vector of magnitude 15 units in the direction of vector $\hat{i}-2\hat{j}+2\hat{k}$.

6. If A is a matrix of order 2 × 3 and B is a matrix of order 3×5, what is the order of matrix (AB)'?

7. Write the vector equation of the following line : $\dfrac{x-5}{3}=\dfrac{y+4}{7}=\dfrac{6-z}{2}$

8. Find x, if $\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix}=\begin{vmatrix} 2x & 4 \\ -5 & 3 \end{vmatrix}$.

9. What is the cosine of the angle which the vector $\sqrt{2}\hat{i}+\hat{j}+\hat{k}$. makes with y-axis?

10. Evaluate: $\int_0^{\pi/4}\sqrt{1+\sin 2x}\ dx$

Section - B

11. Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one - one but not onto.

12. Prove the following : $\tan^{-1} x + \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) = \tan^{-1}\left(\dfrac{3x+x^3}{1-3x^2}\right)$

OR

Prove the following: $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\dfrac{1+x^2}{2+x^2}}$

13. Two schools P and Q decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School P decided to award a total of Rs. 13000 for the three values to 5, 4 and 3 students respectively while school Q decided to award Rs.12700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to Rs.4700, then

(i) Represent the above situation by a matrix equation and form Linear equations using matrix multiplication.

(ii) Is it possible to solve the system of equations so obtained using matrices?

(iii) Which value you prefer to be rewarded most and why?

14. Find all the points of discontinuity of the function $f(x)$ - $[x^2]$ on $[1,2]$, where $[x^2]$ denotes the greatest integer function.

15. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$.

OR

Differentiate, $\sin^{-1}(2x\sqrt{1-x^2})$ w.r.t. $\cos^{-1}\left(\dfrac{1-x^2}{1-x^2}\right)$.

16. Find the intervals in which the following function is strictly increasing or strictly decreasing: $f(x) = 20 - 9x + 6x^2 - x^3$.

OR

For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through origin.

17.   Evaluate: $\int_{-1}^{1/2} |x\cos \pi x| dx$.

18.   Form the differential equation corresponding to the function $y = c(x- c)^2$.

19.   Solve the differential equation: $(x - 1)\dfrac{dy}{dx} = 2xy$, given that $y(2) = 1$.

20.   If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

21.   Show that the four points (0, -1, -1), (4, 5, 1),(3, 9, 4) and (-4, 4, 4) are coplanar. Also, find the equation of the plane containing them.

OR

Find the equation of perpendicular drawn from the point P(2, 4, -1) to the

line $\dfrac{x+5}{1} = \dfrac{y+3}{4} = \dfrac{z-6}{-9}$.

22.   A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six. Write any three benefits of speaking the truth.

Section - C

23.   Using elementary transformations, find the inverse of the matrix: $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$

24.   The sum of the perimeter of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the side of a square is double the radius of the circle.

OR

A helicopter is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). Find the nearest distance between the soldier and the helicopter.

25.   Evaluate: $\int \dfrac{1}{\sin x(5- 4\cos x)} dx$.

OR

Evaluate: $\int \sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx.$

26. Draw a rough sketch of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 1$ integration, find the area of the enclosed region.

27. Find the equation of the plane passing through the point $(1,1,1)$ and perpendicular to each of the following planes $x + 2y + 3z = 7$ and $2x - 3y + 4z = 0$.

28. A chemical industry produces two fertilizers, A and B. The following table gives the units of ingredients C and D (per kg) of fertilizers A and B as well as minimum requirements of C and D, and costs per kg of A and B.

	Fertilizers (in units)		Minimum requirement (in units)
	A	B	
Ingredient C (per kg)	1	2	80
Ingredient D (per kg)	3	1	75
Cost per kg (in ₹)	4	6	

Find the quantities of A and B which would minimize the cost. What are the side effects of using fertilizers.

29. A company has two plants to manufacture bicycles. The first plant manufactures 60% of the bicycles and the second plant, 40%. Also, 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be of standard quality. Find the probability that it comes from the second plant. Would you suggest bicycle as a mode of transport to a student.

OR

In a bolt factory, three machines, A, B, C, manufacture 25%, 35% and 40% of the total production respectively. Of their respective output, 5%, 4% and 2% are defective. A bolt is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by the machine C. What is the value of skill?