FLT-I CLASS 12TH

SOLUTIONS

1. Given R = {(x, y), x, y $\in$ N, x $\leq$ y²}

Let x = 14, y = 4, z = 2

(x, y) $\in$ R as 14 $\leq$ (4)2 $\Rightarrow$ 14 $\leq$ 16, true.

(y, z) $\in$ R as 4 $\leq$ $(2)^2$ $\Rightarrow$ 4 $\leq$ 4, true.

Also, (x, z) $\notin$ R as 14 $\leq$ $(2)^2$ $\Rightarrow$ 14 $\leq$ 4, false.

Hence not transitive.

2. $2\times\frac{\pi}{3}+3\times\frac{\pi}{6}=\frac{7\pi}{6}$

3. If matrix is not invertible

$\begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ = 0 $\Rightarrow$ 2 – k + 15 = 0 $\Rightarrow$ k = 17.

4. $\int \frac{\sec x \tan x}{\sqrt{4-\sec^2 x}}\,dx$

Put sec x = t $\Rightarrow$ sec x tan x dx = dt

$\int \frac{1}{\sqrt{4-t^2}}\,dt = \sin^{-1}\left(\frac{t}{2}\right)+c = \sin^{-1}\left(\frac{\sec x}{2}\right)+c.$

5. Let $\vec{r} = \hat{i} - 2\hat{j} + 2\hat{k}$

Direction of $\vec{r}$ is $\hat{r} = \frac{1}{3}\left(\hat{i}-2\hat{j}+2\hat{k}\right)$

$\therefore$ Vector of magnitude 15 along $\hat{r}$ is

$15\hat{r} = 5\hat{i} - 10\hat{j} + 10\hat{k}$

6. Order of (AB)’ is 5 $\times$ 2.

7. Line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{-2}$

Passes through the point (5, - 4, 6) and direction ratios are 3, 7, – 2

$\therefore$ Vector equation of line is

$$\vec{r} = \left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} - 2\hat{k}\right)$$

8. Consider $\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ -5 & 3 \end{vmatrix}$

$\Rightarrow \quad 6 + 20 = 6x + 20 \quad \Rightarrow \quad 6x = 6 \quad \Rightarrow \quad x = 1.$

9. Let $\vec{r} = \sqrt{2}\hat{i} + \hat{j} + \hat{k}$

$$\Rightarrow \quad \hat{r} = \frac{\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}}{\sqrt{2+1+1}} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$$

Cosine of the angle with y-axis is $\frac{1}{2}$.

10. $\int_0^{\pi/4} \sqrt{1 + \sin 2x}\, dx$

$$\int_0^{\pi/4} \sqrt{\cos^2 x + \sin^2 x + 2\sin x \cos x}\; dx$$

$$= \int_0^{\pi/4} \sqrt{(\cos x + \sin x)^2}\; dx$$

$$= \int_0^{\pi/4} (\cos x + \sin x)\, dx = [\sin x - \cos x]_0^{\pi/4}$$

$$= \left(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right) - (\sin 0 - \cos 0) = 1.$$

11. $f(x) = x^2 + x + 1$

Let $x_1, y_1 \in N$ such that $f(x_1) = f(y_1)$

$\therefore\ x_1^2 + x_1 + 1 = y_1^2 + y_1 + 1$

$\Rightarrow \quad (x_1 - y_1)(x_1 + y_1 + 1) = 0 \qquad$ [As $x_1 + y_1 + 1 \neq 0$ for any N]

$\Rightarrow \quad x_1 = y_1 \quad \Rightarrow \quad$ f is one-one function

Clearly $f(x) = x^2 + x + 1 \geq 3$ for $x \in N$

But f(x) does not assume values 1 and 2

$\therefore\ f : N \to N$ is not onto function.

12. Consider,

$$\tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left[\frac{x + \frac{2x}{1-x^2}}{1 - x\left(\frac{2x}{1-x^2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{x - x^3 + 2x}{1 - x^2 - 2x^2}\right] = \tan^{-1}\left[\frac{3x - x^3}{1 - 3x^2}\right]$$

OR

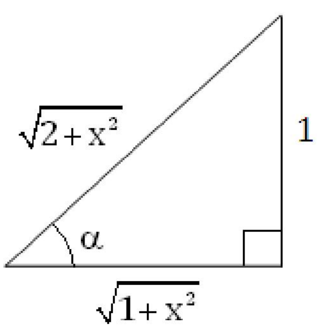
Consider, $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$

Let $x = \cot\,\theta$

$$\Rightarrow \quad \cot\,[\tan^{-1}(\sin\theta)] = \cos\left[\tan^{-1}\left(\frac{1}{\sqrt{1+\cot^2\theta}}\right)\right]$$

$$= \cos\left[\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right] \quad \text{....(i)}$$

Let $\alpha = \tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) \Rightarrow \tan\,\alpha = \frac{1}{\sqrt{1+x^2}}$



$$\Rightarrow \quad \cos\,\alpha = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

$$\Rightarrow \quad \alpha = \cos^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

From (i), $\cos\left[\cos^{-1}\sqrt{\frac{1+x^2}{2+x^2}}\right] = \sqrt{\frac{1+x^2}{2+x^2}}.$

13. (i) $\begin{bmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13000 \\ 12700 \\ 4700 \end{bmatrix}$

$\therefore$ 5x + 4y + 3z = 13000

4x + 3y + 5z = 12700

x + y + z = 4700

(ii) let A = $\begin{bmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore |A| = 5(-2) - 4(-1) + 3\,(1)$

$= -10 + 4 + 3 = -3 \neq 0$

$\because A^{-1}$ exists, so equations have a unique solution.

(iii) Any answer of three values with proper reasoning will be considered correct.

14. Given function is f(x) = [x^2], [1, 2]

i.e., $1 \le x \le 2 \Rightarrow \quad 1 \le x^2 \le 4$

$\therefore$ function is f(x) = [x^2] = $\begin{cases} 1 \quad , & 1 \le x, \sqrt{2} \\ 2 \quad , & \sqrt{2} \le x < \sqrt{3} \\ 3 \quad , & \sqrt{3} \le x < \sqrt{2} \end{cases}$

As within the intervals $\left[1, \sqrt{2}\right), \left[\sqrt{2}, \sqrt{3}\right)$ and $\left[\sqrt{3}, 2\right)$ function is constant, here continuous.

Let us check for the points $x = \sqrt{2}$ and $x = \sqrt{3}$

For $x = \sqrt{2}$

L. H. L. = $\underset{x \to \sqrt{2}^-}{\text{Lt}} f(x) = \underset{h \to 0}{\text{Lt}} f\left(\sqrt{2} - h\right) = \underset{h \to 0}{\text{Lt}} [1] = 1$

R. H. L. = $\underset{x \to \sqrt{2}^+}{\text{Lt}} f(x) = \underset{h \to 0}{\text{Lt}} f\left(\sqrt{2} + h\right) = \underset{h \to 0}{\text{Lt}} [2] = 2$

As L. H. L $\neq$ R. H. L.

$\therefore \underset{x\to\sqrt{2}}{\text{Lt}} f(x)$ does not exist.

Hence function is discontinuous at x = $\sqrt{2}$.

For $x = \sqrt{3}$

L. H. L. = $\underset{x\to\sqrt{3}^-}{\text{Lt}} f(x) = \underset{h\to 0}{\text{Lt}} f\left(\sqrt{3}-h\right) = \underset{h\to 0}{\text{Lt}} [2] = 2$

R. H. L. = $\underset{x\to\sqrt{3}^+}{\text{Lt}} f(x) = \underset{h\to 0}{\text{Lt}} f\left(\sqrt{3}+h\right) = \underset{h\to 0}{\text{Lt}} [3] = 3$

As L. H. L $\neq$ R. H. L.

$\therefore \underset{x\to\sqrt{3}}{\text{Lt}} f(x)$ does not exist.

Hence function is discontinuous at $x = \sqrt{3}$

$\therefore$ Point of discontinuity are $x = \sqrt{2}, x = \sqrt{3}$.

15. Consider $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

Let x = $\sin\theta$, y = $\sin\phi$ $\Rightarrow$ $\theta = \sin^{-1}x, \phi = \sin^{-1}y$ ...(i)

$\Rightarrow \quad \sqrt{1-\sin^2\theta} + \sqrt{1-\sin^2\phi} = a(\sin\theta - \sin\phi)$

$\Rightarrow \quad \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$\Rightarrow \quad 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$

$= a\left(2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)\right)$

$\Rightarrow \quad \cot\left(\frac{\theta-\phi}{2}\right) = a$

$\Rightarrow \quad \theta - \phi = 2\cot^{-1}a$

$\Rightarrow \quad \sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$

Differentiating w. r. t. x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}.$$

OR

Let $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ and $t = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

We have to find $\frac{dy}{dt}$.

Consider $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Let $x = \sin\theta \quad \Rightarrow \quad \theta = \sin^{-1} x$

$y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$

$= \sin^{-1}\left(2\sin\theta\cos\theta\right) = \sin^{-1}\left(\sin 2\theta\right) = 2\theta$

$\Rightarrow \quad y = 2\sin^{-1} x \quad \therefore \quad \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$ ...(i)

Consider $t = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1} x$

$\therefore \frac{dt}{dx} = 2\times\frac{1}{1+x^2} = \frac{2}{1+x^2}$ ...(ii)

Now $\frac{dy}{dt} = \frac{dy}{dx}\times\frac{dx}{dt} = \frac{2}{\sqrt{1-x^2}}\times\frac{1+x^2}{2} = \frac{1+x^2}{\sqrt{1-x^2}}$

16. Consider $f(x) = 20 - 9x + 6x^2 - x^3$

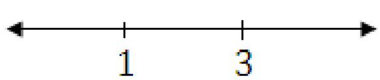
$f'(x) = -9 + 12x - 3x^2 = 3(x^2 - 4x + 3)$

$= -3(x-1)(x-3)$ ...(i)

For turning points $f'(x) = 0$

$\Rightarrow \quad (x-1)(x-3) = 0 \Rightarrow \quad x = 1, 3$

From (i)



For $x < 1$, $f'(x) = (-)\,(-)\,(-)$

$\Rightarrow \quad f'(x) < 0 \Rightarrow$ function is $\downarrow$.

For $1 < x < 3$, $f'(x) = (-)\,(+)\,(+)$

$\Rightarrow \quad f'(x) < 0 \Rightarrow$ function is $\downarrow$.

Hence functions increases for (1, 3) and function decreases for $(-\infty, 1) \cup (3, \infty)$.

OR

Let the point (x_1, y_1) be on the curve $y = 4x^3 - 2x^5$ ...(i)

Tangent passes through origin.

As point (x_1, y_1) lies on curve (i)

$\therefore\ y_1 = 4x_1^3 - 2x_1^5$ ...(ii)

Differentiating (i) w. r. t. x, we get

$$\frac{dy}{dx} = 12x^2 - 10x^4$$

$$\left.\frac{dy}{dx}\right]_{(x_1, y_1)} = 12x_1^2 - 10x_1^4 \qquad \text{[Slope of the tangent]}$$

$\therefore$ Equation of the tangent at (x_1, y_1) is

$y_1 - y_1 = \left(12x_1^2 - 10x_1^4\right)\left(0 - x_1\right)$

If this tangent passes through origin (0, 0) then, $0 - y_1 = \left(12x_1^2 - 10x_1^4\right)\left(0 - x_1\right)$

$\Rightarrow \quad 2x_1^5 - 4x_1^3 = -12x_1^3 + 10x_1^5$ [From (ii)]

$\Rightarrow \quad -8x_1^5 - 8x_1^3 = 0 \quad \Rightarrow \quad -8x_1^3\left(x_1^2 - 1\right) = 0$

$\Rightarrow \quad x_1^3 = 0 \quad$ or $\quad x_1^2 - 1 = 0 \quad \Rightarrow \quad x_1 = 0, 1, -1$

From (ii),

when $x_1 = 0$, $y_1 = 0 - 0 = 0$ Point is (0, 0)

when $x_1 = 1$ $y_1 = 4 - 2 = 2$ Point is (1, 2)

when $x_1 = -1$, $y_1 = -4 + 2 = -2$, Point is (−1, − 2)

17. Consider $\int_{-1}^{1/2} \left| x \cos \pi x \right| dx$

We have $-1 < x < \frac{1}{2} \Rightarrow -\pi < \pi x < \frac{\pi}{2}$

$\cos \pi x = 0 \Rightarrow \pi x = = -\frac{\pi}{2} \Rightarrow x = -\frac{1}{2}$

For $-1 < x - \frac{1}{2}$, $x < 0$ and $\cos \pi x < 0 \Rightarrow x \cos \pi x > 0$.

For $-\frac{1}{2} < x < 0$, $x < 0$ and $\cos \pi x > 0 \Rightarrow x \cos \pi x < 0$.

For $0 < x < \frac{1}{2}$, $x > 0$ and $\cos \pi x > 0 \Rightarrow x \cos \pi x > 0$.

Consider $\int x \cos \pi x \, dx = x.\frac{\sin \pi x}{\pi} - \int 1.\frac{\sin \pi x}{\pi} dx$

$= \frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x$

$\therefore \int_{-1}^{1/2} \left| x \cos \pi x \right| dx = \int_{-1}^{1/2} x \cos \pi x \, dx + \int_{-1/2}^{0} (-x \cos \pi x) \, dx + \int_{0}^{1/2} (x \cos \pi x) \, dx.$

$$= \left\{ \frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x \right\}_{1}^{-1/2} - \left\{ \frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x \right\}_{-1/2}^{0} + \left\{ \frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x \right\}_{0}^{1/2}$$

$$= \left\{ -\frac{1}{2\pi} \sin\left(-\frac{\pi}{2}\right) + \frac{1}{\pi^2} \cos\left(-\frac{\pi}{2}\right) \right\} - \left\{ -\frac{1}{\pi} \sin(-\pi) + \frac{1}{\pi^2} \cos(-\pi) \right\} - \left\{ 0 \sin 0 + \frac{1}{\pi^2} \cos 0 \right\} +$$

$$\left\{ -\frac{1}{2\pi} \sin\left(-\frac{\pi}{2}\right) + \frac{1}{\pi^2} \cos\left(-\frac{\pi}{2}\right) \right\} - \left\{ \frac{1}{2\pi} \sin\frac{\pi}{2} + \frac{1}{\pi^2} \cos\frac{\pi}{2} \right\} - \left\{ 0 \sin 0 + \frac{1}{\pi^2} \cos 0 \right\}$$

$$= \left[-\frac{1}{2\pi} \times -1 + 0 \right] - \left[0 + \frac{1}{\pi^2} \times -1 \right] - \left[0 + \frac{1}{\pi^2} \times 1 \right] + \left[\frac{-1}{2\pi} \times -1 + 0 \right] + \left[\frac{1}{2\pi} \times 1 + 0 \right] - \left[0 + \frac{1}{\pi^2} \times 1 \right]$$

$$= \frac{1}{2\pi} + \frac{1}{\pi^2} - \frac{1}{\pi^2} + \frac{1}{2\pi} - \frac{1}{\pi^2} = \frac{3}{2\pi} - \frac{1}{\pi^2}$$

18. Consider $y = c(x - c^2)$ ....(i)

Differentiating (i) w. r. t. x, we get $y' = 2c(x - c)$ ...(ii)

Dividing (i) by (ii), we get

$$\frac{y}{y'} = \frac{x-c}{2} \Rightarrow \frac{2y}{y'} = x - c \Rightarrow c = x - \frac{2y}{y'}.$$

Substituting for c in (ii), we get

$$y' = 2\left(x - \frac{2y}{y'}\right)\left(\frac{2y}{y'}\right) \Rightarrow \quad y' = \frac{xy' - 2y}{y'} \cdot \frac{4y}{y'}$$

$$\Rightarrow \quad y'^3 = 4xyy' - 8y^2$$

$\Rightarrow \quad y'^3 = -4xyy' + 8y^2 = 0$ is the required differential equation.

19. Consider $(x-1)\frac{dy}{dx} = 2xy \Rightarrow \quad \frac{dy}{y} = \frac{2x}{x-1}dx,$ Integrating both sides, we get

$$\int \frac{dy}{y} = 2\int \frac{x}{x-1}dx = 2\int \left(x + \frac{1}{x-1}\right)dx$$

$$\Rightarrow \quad \log|y| = 2(x + \log|x-1|) + c \qquad \text{...(i)}$$

Given, y = 1, when x = 2

$\Rightarrow \quad \log\ 1 = 2(2 + \log 1) + c \Rightarrow \quad c = -4$

Substituting in (i), we get $\log|y| = 2(x + \log|x-1|) - 4$ is the required solution.

20. Let $\hat{a}$ and $\hat{b}$ be two unit vectors, such that $|\hat{a} + \hat{b}| = 1.$ Also $|\hat{a}| = 1, |\hat{b}| = 1$

Consider $|\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b})^2 = \widehat{a^2} + \widehat{b^2} - 2\hat{a}.\hat{b}$ ..(i)

Also $|\hat{a} + \hat{b}| = 1 \Rightarrow |\hat{a} + \hat{b}|^2 = 1 \Rightarrow (\hat{a} + \hat{b})^2 = 1$

$\Rightarrow \widehat{a^2} + \widehat{b^2} + 2\hat{a}.\hat{b} = 1 \Rightarrow |\hat{a}|^2 + |\hat{b}|^2 + 2\hat{a}.\hat{b} = 1$

$\Rightarrow 1 + 1 + 2\hat{a}.\hat{b} = 1 \Rightarrow \quad 2\hat{a}.\hat{b} = -1$ ...(iii)

From (i) and (ii), we get

$$|\hat{a} - \hat{b}|^2 = (1)^2 + (1)^2 - (-1) = 3 \Rightarrow |\hat{a} - \hat{b}| = \sqrt{3}.$$

21. Let us find equation of the plane through three points say (0, −1, −1), (4, 5, 1) and (3, 9, 4)

Equation is $\begin{vmatrix} x-0 & y+1 & z+1 \\ 4-0 & 5+1 & 1+1 \\ 3-0 & 9+1 & 4+1 \end{vmatrix} = 0$

$\Rightarrow \quad \begin{vmatrix} x & y+1 & z+1 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix} = 0$

$\Rightarrow$ x(30 – 20) – (y + 1) (20 – 6) + (z + 1) (40 – 18) = 0

$\Rightarrow$ 10x – 14y – 14 + 22z + 22 = 0

$\Rightarrow$ 10x – 4y + 22z + 8 = 0 or 5x – 7y + 11z + 4 = 0 is equation of a plane.

If four points are coplanar then (–4, 4, 4) also must lie on a plane, i.e.

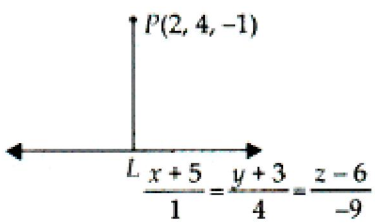
–20 – 28 + 44 + 4 = 0, True

Hence points are coplanar.

OR

Let PL be perpendicular from P (2, 4, –1) on line

$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$.



General point on the line is $L(\lambda - 5, 4\lambda - 3, -9\lambda + 6)$

Direction ratios of PL are $\lambda - 5 - 2, 4\lambda - 3 - 4, -9\lambda + 6 + 1$.

i.e., < λ – 7, 4λ – 7, –9λ + 7 > ...(i)

If PL is perpendicular to the given line then 1(λ – 7) + 4 (4λ – 7) – 9(–9λ + 7) = 0

98λ-98=0

λ=1

Substituting λ=1in (i), we get

Direction ratios of perpendicular PL as < −6, −3, −2 > or <6, 3, 2 >

∴ Equation of perpendicular is $\frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$.

22. In a throw of a die, let

E_1 = event of getting a six,

E_2 = event of not getting a six,

E = event that the man reports that it is a six.

Then, $P(E_1)=\frac{1}{6}$, and $P(E_2)=\left(1-\frac{1}{6}\right)=\frac{5}{6}$.

$P(E/E_1)$ = probability that the man reports that six occurs, when six has actually occurred

= probability that the man speaks the truth = $\frac{3}{4}$.

$P(E/E_2)$ = probability that the man reports that six occurs, when six has not actually occurred.

= probability that the man does not speak the truth

= $\left(1-\frac{3}{4}\right)=\frac{1}{4}$.

Probability of getting a six, given that the man reports it to be six

= $P(E_1/E)=\frac{P(E/E_1).P(E_1)}{P(E/E_1).P(E_1)+P(E/E_2).P(E_2)}$ [by Bayes' theorem]

= $\frac{\left(\frac{3}{4}\times\frac{1}{6}\right)}{\left(\frac{3}{4}\times\frac{1}{6}\right)+\left(\frac{1}{4}\times\frac{5}{6}\right)}=\left(\frac{1}{8}\times 3\right)=\frac{3}{8}$.

Hence, the required probability is $\frac{3}{8}$.

The three benefits are

(1) It gives positive thinking and satisfaction

(2) Everyone loves it

(3) It is good life skill.

23. Let $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$

Consider A = IA

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} A$$

[By performing $R_3 \to R_3 - 3R_1$]

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 3 & 1 & -1 \\ -3 & 0 & 1 \end{pmatrix} A$$

[By performing $R_1 \to R_1 + R_3$ and $R_2 \to R_2 - R_3$]

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 3 & 1 & -1 \\ -6 & -1 & 2 \end{pmatrix} A$$

[By performing $R_3 \to R_3 - R_2$]

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ -6 & -1 & 2 \end{pmatrix} A$$

[By performing $R_2 \to R_2 - R_3$]

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix} A$$

[By performing $R_3 \to (-1)\, R_3$]

Hence $A^{-1} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$.

24. Let r be the radius of a circle and x be side of a square. Given $2\pi r + 4x = k$ ..(i)

Sum of areas (A) = $\pi r^2 + x^2 = \pi\left(\frac{k-4x}{2\pi}\right)^2 + x^2$ [From (i)]

$\Rightarrow \quad A = \frac{1}{4\pi}(k-4x)^2 + x^2$

$\frac{dA}{dx} = \frac{1}{4\pi} \cdot 2(k-4x)(-4) + 2x = \frac{2(4x-k)}{\pi} + 2x$ ...(ii)

For minimum A, $\frac{dA}{dx} = 0$

$\Rightarrow \quad \frac{2(4x-k)}{\pi} + 2x = 0 \Rightarrow 8x - 2k + 2\pi x = 0$

$\Rightarrow \quad (4+\pi)\, x = k \Rightarrow \quad x = \frac{k}{4+\pi}$

$\frac{d^2A}{dx^2} = \frac{2}{\pi}(4) + 2 \Rightarrow \left.\frac{d^2A}{dx^2}\right]_{x=\frac{k}{4+\pi}} = \frac{8}{\pi} + 2 > 0$ [From (ii)]

Hence area is minimum for x = $\frac{k}{4+\pi}$

$\Rightarrow \quad 4x + \pi x = 2\pi r + 4x \Rightarrow \pi x = 2\pi r \Rightarrow x = 2r$ [From (i)]

$\Rightarrow$ side of a square is double the radius of the circle.

OR

Let point (x, y) on the curve $y = x^2 + 2$ ...(i)

Position of helicopter is nearest to the soldier at (3, 2)

Distance D = $\sqrt{(x-3)^2 + (y-2)^2}$

If D is minimum, thenD^2 is minimum

$\therefore\ D^2 = E = (x-3)^2 + (y-2)^2 = (x-3)^2 + (x^2+2-2)^2$

$= (x-3)^2 + x^4$ ....[From (i)]

$$\frac{dE}{dx} = 2(x-3) + 4x^3 = 2[2x^3 + x - 3]$$

For minimum E, $\frac{dE}{dx} = 0 \Rightarrow 2x^3 + x - 3 = 0$, we notice for x = 1, $\frac{dE}{dx} = 0$

$$\Rightarrow \quad \frac{dE}{dx} = 2(x-1)(2x^2+2x+3) = 0 \Rightarrow x - 1 = 0$$

or $\quad 2x^2 + 2x + 3 = 0$

$\Rightarrow \quad$ x = 1 only as there is no real solution for $2x^2 + 2x + 3 = 0$

$$\frac{d^2E}{dx^2} = 2[6x^2+1] \Rightarrow \left.\frac{d^2E}{dx^2}\right]_{x=1} > 0;$$

$\therefore$ for x = 1, distance is minimum

Substituting in curve (i), we get, y = 1 + 2 + 3.

$\therefore$ Point (1, 3) is nearest to the point (3, 2)

$\therefore$ Minimum distance $= \sqrt{(3-1)^2 + (2-3)^2} = \sqrt{5}$.

25. Consider $\int \frac{dx}{\sin x(5-4\cos x)}$

Multiplying and dividing by sin x, we get

$$= \int \frac{dx}{\sin^2 x(5-4\cos x)} \qquad \begin{pmatrix} \text{Let} & \cos x = t \\ \Rightarrow -\sin x dx = dt \end{pmatrix}$$

$$= -\int \frac{dt}{(1-t^2)(5-4t)}$$

$$= -\int \frac{dt}{(1-t)(1+t)(5-4t)} \qquad ...(i)$$

Consider $\frac{1}{(1-t)(1+t)(5-4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t}$

$$\Rightarrow \quad \frac{1}{(1-t)(1+t)(5-4t)}$$

$$= \frac{A(1+t)(5-4t)+B(1-t)(5-4t)+C(1-t)(1+t)}{(1-t)(1+t)(5-4t)}$$

$\Rightarrow \quad 1 = A(5 + t - 4t^2) + B(5 - 9t + 4t^2) + C(1 - t^2)$

$= (5A + 5B + C) + t(A - 9B) + t^2(-4A + 4B - C)$

Comparing the coefficients, we get

$5A + 5B + C = 1$ ...(ii)

$A - 9B = 0$ ...(iii)

$-4A + 4B - C = 0$ ...(iv)

Substituting from (iii) for A in (ii) and (iv), we get

$$\left.\begin{matrix} 50B + C = 1 \\ -32B - C = 0 \end{matrix}\right] \Rightarrow B = \frac{1}{18}, C = -\frac{16}{9}$$

From (iii), $A = \frac{1}{2}$

$\therefore$ From (i),

$$\int \frac{dx}{\sin x(5-4\cos x)} = -\int \frac{dt}{(1-t)(1+t)(5-4t)}$$

$$= -\left(\int \frac{\frac{1}{2}}{1-t}\,dt + \int \frac{\frac{1}{18}}{1+t}\,dt + \int \frac{-\frac{16}{9}}{5-4t}\,dt \right)$$

$$= -\frac{1}{2}\int \frac{dt}{1-t} - \frac{1}{18}\int \frac{dt}{1+t} + \frac{16}{9}\int \frac{dt}{5-4t}$$

$$= -\frac{1}{2}.\frac{\log|1-t|}{-1} - \frac{1}{18}\log|1+t| + \frac{16}{9}\int \frac{dt}{5-4t} + c$$

$$= -\frac{1}{2}\log|1-\cos x| - \frac{1}{18}\log|1+\cos x| - \frac{4}{9}\log|5-4\cos x| + c.$$

OR

$$\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx \quad \left(\begin{array}{l}\text{Put } \sqrt{x}=\cos\theta \Rightarrow x=\cos^2\theta \\ \qquad\qquad\quad \Rightarrow dx=-2\sin\theta\cos\theta d\theta\end{array}\right)$$

$$= -2\int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}.\sin\theta\cos\theta d\theta$$

$$= -2\int\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}.\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cos\theta d\theta$$

$$= -2\int\left(2\sin^2\frac{\theta}{2}\right)\cos\theta d\theta = -2\int(1-\cos\theta)\cos\theta d\theta$$

$$= -2\int\left(\cos\theta-\cos^2\theta\right)d\theta = -2\left(\cos\theta-\frac{1+\cos 2\theta}{2}\right)d\theta$$

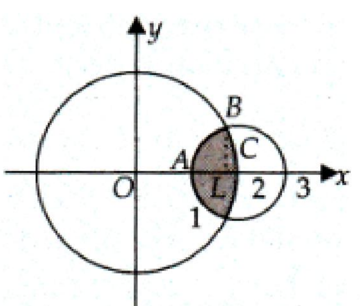
$$= -2\int(2\cos\theta-1-\cos 2\theta)d\theta = -\left[2\sin-\theta-\frac{\sin 2\theta}{2}\right]+c$$

$$= -2\sin\theta+\theta+\sin\theta\cos\theta+c$$

$$= -2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x}\sqrt{1-x}+c.$$

26. Given circles are $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 1$

On plotting the circles, we notice, we have to find the shaded area.



We notice both the curves are symmetrical to x-axis, as both functions are even with respect to y.

$\therefore$ Area = 2 × Area in

Ist. quadrant = 2ar(ABC)

= 2[ar(ABL) + ar(LBC)] ...(i)

Eliminating 'y' from the two equations, we get

$4 - x^2 = 1 - (x-2)^2 \Rightarrow 4 - x^2 = 1 - x^2 + 4x - 4$

$\Rightarrow \quad 4x = 7 \Rightarrow x = \frac{7}{4}$

For ar(ABL) : curve is $(x-2)^2 + y^2 = 1$, x-axis, $x = 1$ and $x = \frac{7}{4}$.

$$\therefore \text{ar(ABL)} = \int_1^{7/4} \sqrt{1-(x-2)^2}\,dx$$

$$= \left[\frac{x-2}{2}\sqrt{1-(x-2)^2} + \frac{1}{2}\sin^{-1}(x-2)\right]_1^{\frac{7}{4}}$$

$$= \left\{-\frac{1}{8}\sqrt{\frac{15}{16}} + \frac{1}{2}\sin^{-1}\left(-\frac{1}{4}\right)\right\} - \left\{-\frac{1}{2}\sqrt{0} + \frac{1}{2}\sin^{-1}(-1)\right\}$$

$$= -\frac{\sqrt{15}}{32} - \frac{1}{2}\sin^{-1}\left(\frac{1}{4}\right) + \frac{\pi}{4} \qquad \text{..(ii)}$$

For ar(LBC) : curve is $x^2 + y^2 = 4$, x-axis, $x = \frac{7}{4}$, $x = 2$

$$\therefore \text{ar(LBC)} = \int_{7/4}^{2} \sqrt{4-x^2}\,dx = \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{7/4}^{2}$$

$$= \{2\sqrt{0} + 2\sin^{-1}\} - \left\{\frac{7}{8}\sqrt{\frac{15}{16}} + 2\sin^{-1}\frac{7}{8}\right\}$$

$$= 2.\frac{\pi}{2} - \frac{7\sqrt{15}}{32} - 2\sin^{-1}\frac{7}{8} \qquad \text{..(iii)}$$

Substituting from (ii) and (iii) in (i), we get

$$\text{Area} = 2\left[-\frac{\sqrt{15}}{32} - \frac{1}{2}\sin^{-1} + \frac{\pi}{4} + \pi - \frac{7\sqrt{15}}{32} - 2\sin^{-1}\frac{7}{8}\right]$$

$$= 2\left[\frac{5\pi}{4} - \frac{\sqrt{15}}{4} - \frac{1}{2}\sin^{-1}\frac{1}{4} - 2\sin^{-1}\frac{7}{8}\right]$$

$$= \left[\frac{5\pi}{2} - \frac{\sqrt{15}}{2} - \sin^{-1}\frac{1}{4} - 4\sin^{-1}\frac{7}{8}\right] \text{ sq. units}$$

27. Any plane through (1, 1, 1) is

$a(x - 1) + b(y - 1) + c(z - 1) = 0$ ..(i)

Now (i) being perpendicular to each of the planes $x + 2y + 3z = 7$ and $2x - 3y + 4z = 0$, we have

$a \times 1 + b \times 2 + c \times 3 = 0 \Rightarrow a + 2b + 3c = 0$ ..(ii)

$a \times 2 + b \times (-3) + c \times 4 \Rightarrow 2a - 3b + 4c = 0$ ..(iii)

Cross multiplying (ii) and (iii), we get

$$\frac{a}{(8+9)} = \frac{b}{(6-4)} = \frac{c}{(-3-4)} = k \text{ (say)}$$

$$\Rightarrow \quad \frac{a}{17} = \frac{b}{2} = \frac{c}{-7} = k$$

$\Rightarrow$ $a = 17k$, $b = 2k$ and $c = -7k$.

Putting these values in (i), we get

$17k(x - 1) + 2k(y - 1) - 7(z - 1) = 0$

$\Leftrightarrow$ $17(x - 1) + 2(y - 1) - 7(z - 1) = 0$

$\Leftrightarrow$ $17x + 2y - 7z - 12 = 0$,

which is the required equation of the plane.

28. Let x kg of A and y kg of B be produced.

Then, $x \geq 0$, $y \geq 0$, $x + 2y \geq 80$ and $3x + y \geq 75$.

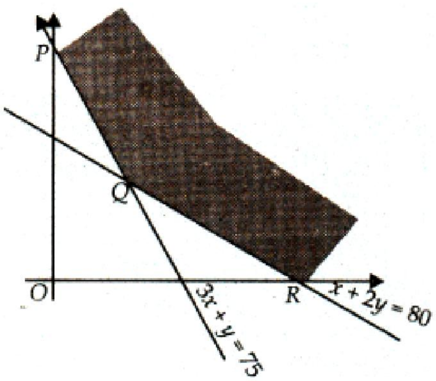
Then cost function is given by $Z = 4x + 6y$.

Thus, we have to minimize $Z = 4x + 6y$, subject to the constraints.

$x \geq 0$, $y \geq 0$, $x + 2y \geq 80$ and $3x + y \geq 75$.

Draw the graphs of the lines

$x = 0$, $y = 0$, $x + 2y = 80$ and $3x + y = 75$.



Since (0, 0) does not satisfy the inequation $x + 2y \geq 80$, the line $x + 2y = 80$ together with the region not containing (0, 0), represents $x + 2y \geq 80$.

Since (0, 0) does not satisfy the inequation $3x + y \geq 75$, the line $3x + y = 75$ together with the region not containing (0, 0) represents $3x + y \geq 75$.

Thus, the shaded region is the feasible region.

The vertices of this region are P, Q and R.

On solving $x = 0$ and $3x + y = 75$, we get the point P(0, 75).

On solving $x + 2y = 80$ and $3x + y = 75$, we get the point Q(14, 33).

On solving $y = 0$ and $x + 2y = 80$, we get the point R(80, 0).

The values of $Z = 4x + 6y$ at the points P(0, 75), Q(14, 33) and (80, 0) are 450, 254 and 320 respectively.

Thus, Z is minimum at Q(14, 33).

Hence , for a minimum cost, 14 kg of A and 33 kg of B must be taken.

Exessive use of fertilizers can spoil the quality of crop.

Also it may cause various diseases.

29. Let E_1 and E_2 be the events of choosing a bicycle from the first plant and the second plant respectively.

Then, $\frac{60}{100} = \frac{3}{5}$, and $P(E_2) = \frac{40}{100} = \frac{2}{5}$.

Let E be the even of choosing a bicycle of standard quality. Then.

$P(E/E_1)$ = probability of choosing a bicycle of standard quality, given that it is produced by the first plant

$= \frac{80}{100} = \frac{4}{5}$.

$P(E/E_2)$ = probability of choosing a bicycle of standard quality, given that it is produced by the second plant = $\frac{90}{100} = \frac{9}{10}$.

The required probability

$P(E_2/E)$ = probability of choosing a bicycle form the second plant, given that it is of standard quality

$= \frac{P(E_2).P(E/E_2)}{P(E_1).P(E/E_1)+P(E_2).P(E/E_2)}$ [by Bayes' theorem]

$$= \frac{\left(\frac{2}{5}\times\frac{9}{10}\right)}{\left(\frac{3}{5}\times\frac{4}{5}\right)+\left(\frac{2}{5}\times\frac{9}{10}\right)} = \frac{3}{7}.$$

Cycle should be used because it is good for health. It causes no pollution and does not use petrol.

OR

Let E_1, E_2 and E_3 be the events of drawing a bolt produced by machine A, B and C respectively. Then,

$$P(E_1) = \frac{25}{100} = \frac{1}{4}, P(E_2) = \frac{35}{100} = \frac{7}{20} \text{ and } P(E_3) = \frac{40}{100} = \frac{2}{5}.$$

Let E be the event of drawing a defective bolt. Then, $P(E/E_1)$ = probability of drawing a defective bolt, given that it is produced by the machine A = $\frac{5}{100} = \frac{1}{20}$.

$P(E/E_2)$ = probability of drawing a defective bolt, given that it is produced by the machine B = $\frac{4}{100} = \frac{1}{25}$.

$P(E/E_3)$ = probability of drawing a defective bolt, given that it is produced by the machine $C = \frac{2}{100} = \frac{1}{50}$.

Probability that the bolt drawn is manufactured by C, given that it is defective

= $P(E_3/E)$

$$= \frac{P(E/E_3).P(E_3)}{P(E/E_1).P(E_1)+P(E/E_2).P(E_2)+P(E/E_3).P(E_3)} \qquad \text{[by Baye's theorem]}$$

$$= \frac{\left(\frac{1}{50}\times\frac{2}{5}\right)}{\left(\frac{1}{20}\times\frac{1}{4}\right)+\left(\frac{1}{25}\times\frac{7}{20}\right)+\left(\frac{1}{50}\times\frac{2}{5}\right)} = \left(\frac{1}{125}\times\frac{2000}{69}\right) = \frac{16}{69}.$$

Hence, the required probability is $\frac{16}{69}$.

Skilled factory can complete the work in better way.